Longest Path in Graphs: 90's and 00's

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IMSc and University of Bergen,
RAA 2017, St. Petersburg

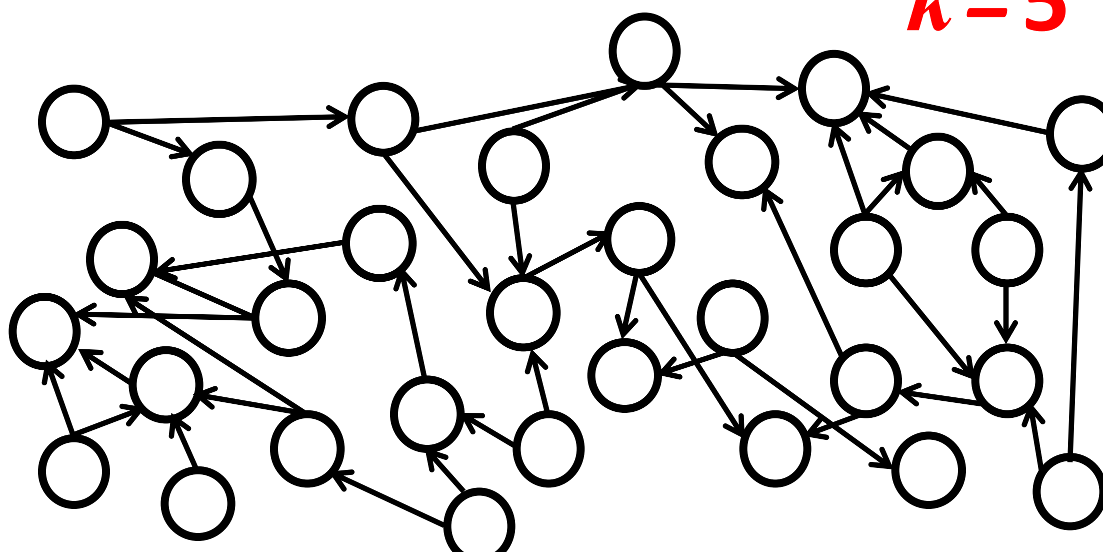
k-Path

k-Path.

Input: Directed graph G; parameter k.

Question: Does G have a path on at least k vertices?

$k = 5$



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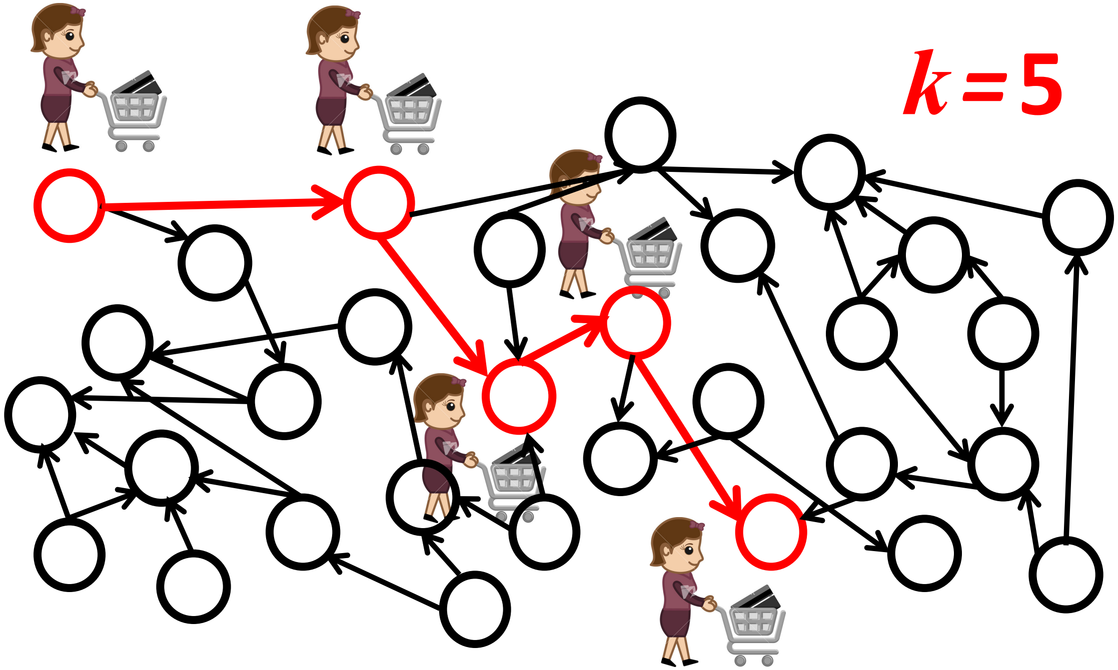

k-Path

Deterministic

$O^*(k!)$	Monien '85
$O^*(k!2^k)$	Bodlaender '93
$O^*((2e)^{k+o(k)})$	Alon, Yuster and Zwick, '95
$O^*(4^{k+o(k)})$	Chen, kneis, Lu, Molle, Richter, Rossmanith, Sze and Zhang, '09
$O^*(2.851^k)$ $[O^*(2.619^k)]$	Fomin, Lokshtanov, Panolan and Saurabh, '16 [-“-, Shachnai and Zehavi, '14]
$O^*(2.597^k)$	Zehavi, '15

k-Path

Deterministic

Other problems: 3-Set k-Packing, 3D k-Matching, subcases of Subgraph Isomorphism, Graph Motif, Partial Cover , k-Internal Out-Branching, ...

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$O^*(2.851^k)$ [$O^*(2.619^k)$]	Fomin, Loks
$O^*(2.597^k)$	Zehavi, '15

Mixing: $O^*(12.155^k)$ to $O^*(8.097^k)$.

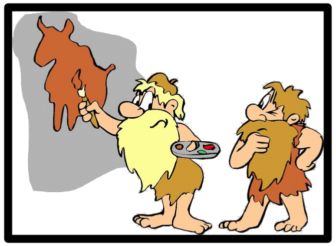
k-Path

Randomized

$O^*(2^k)$	Koutis and Williams, '09
$O^*(1.657^k)$	Björklund, Husfeldt, Kaski and Koivisto, '10 (Undirected)

(Art?) Tutorial

1. Brute-Force
2. Highlights
3. Color Coding
4. Divide-and-Color
5. Representative Sets
6. Mixing



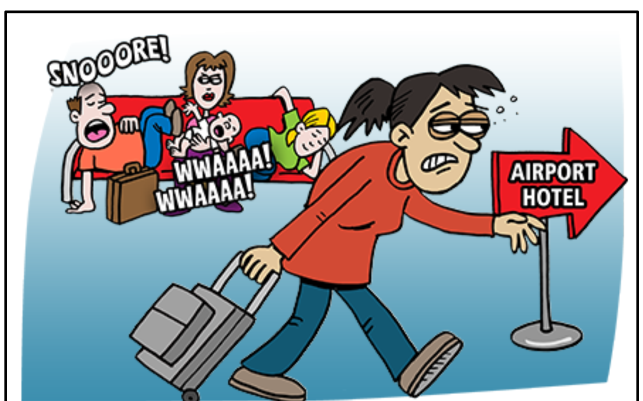
k-Path: Brute-Force

k-Path.

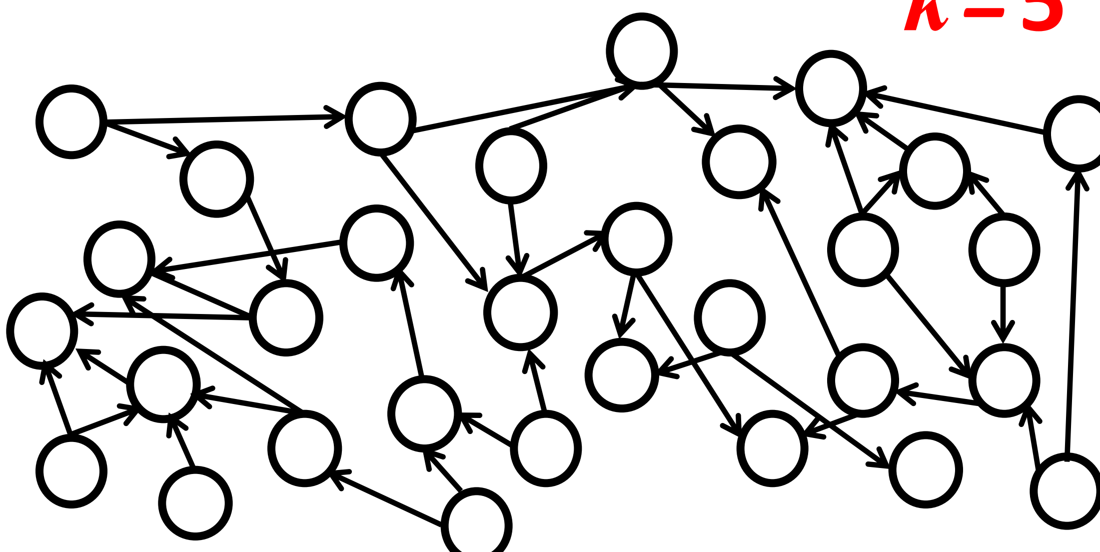
Input: Directed graph G; parameter k.

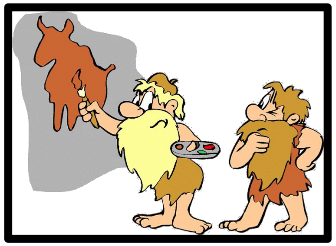
Question: Does G have a path on at least k vertices?

Time: $\binom{n}{k} \cdot k!$



$k = 5$





k-Path: Brute-Force

k-Path.

Input: Directed graph G; parameter k.

Que … ces?

How can we easily identify a solution?

5

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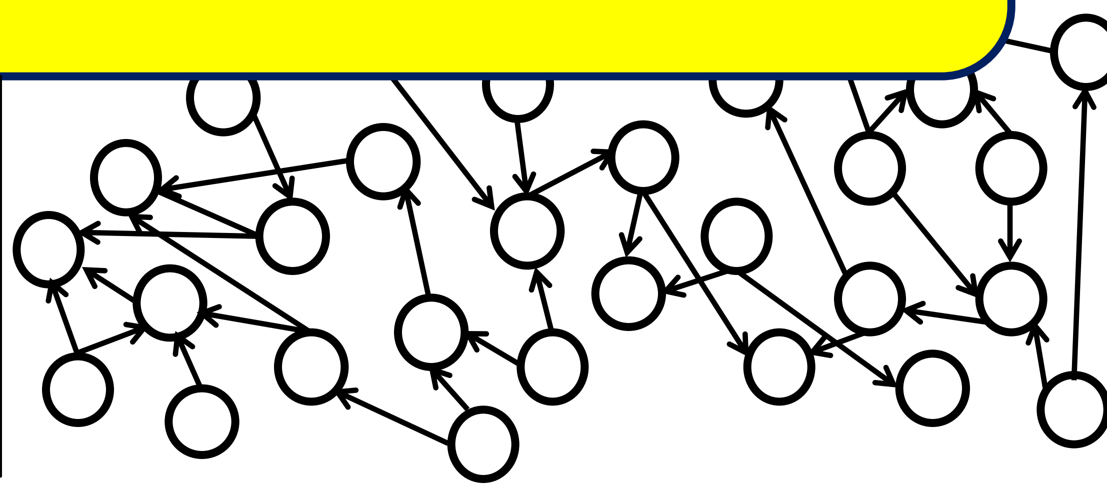
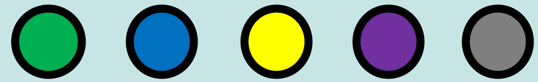


(Art?) Tutorial

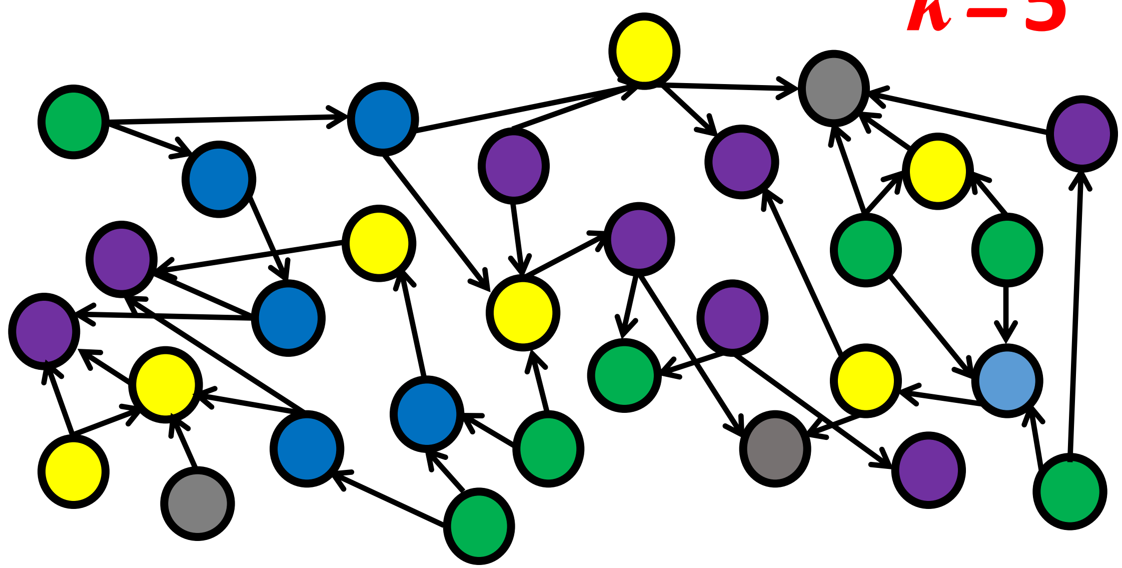
1. **Brute-Force**
2. **Highlights**
3. **Color Coding**
4. **Divide-and-Color**
5. **Representative Sets**
6. **Mixing**

k-Path: Highlights

Color-set: $\{1,2,\ldots,k\}$. 

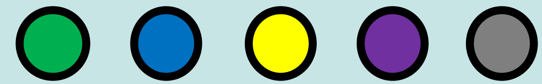
To each vertex, randomly assign a color.

$k=5$



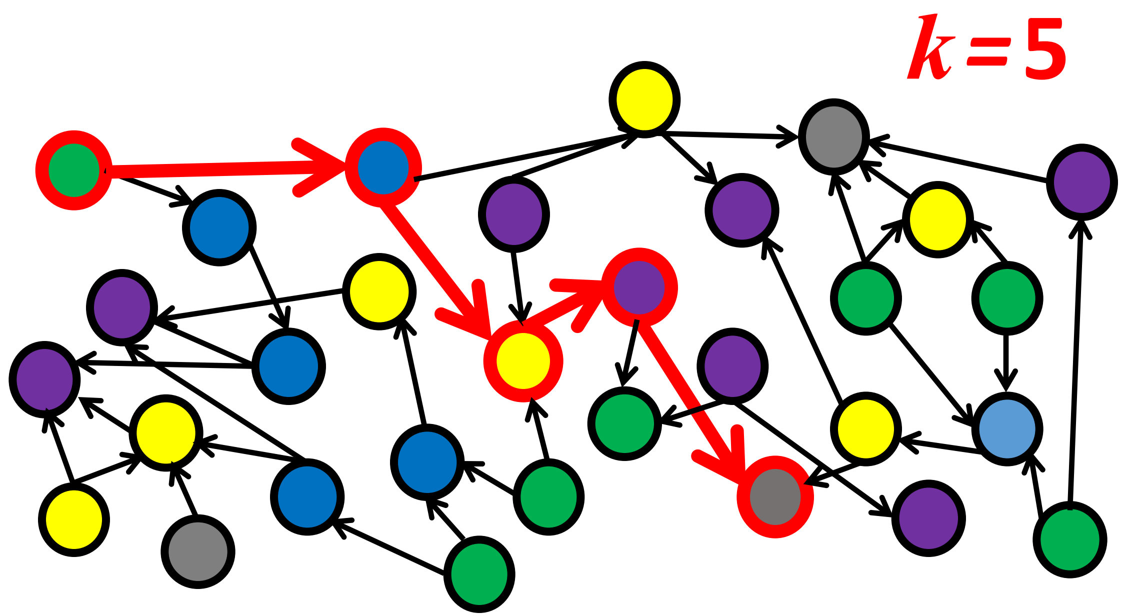
k-Path: Highlights

Color-set: $\{1,2,\ldots,k\}$.

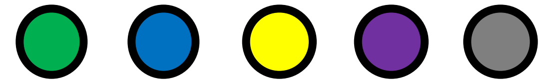


To each vertex, randomly assign a color.

Highlight a solution.



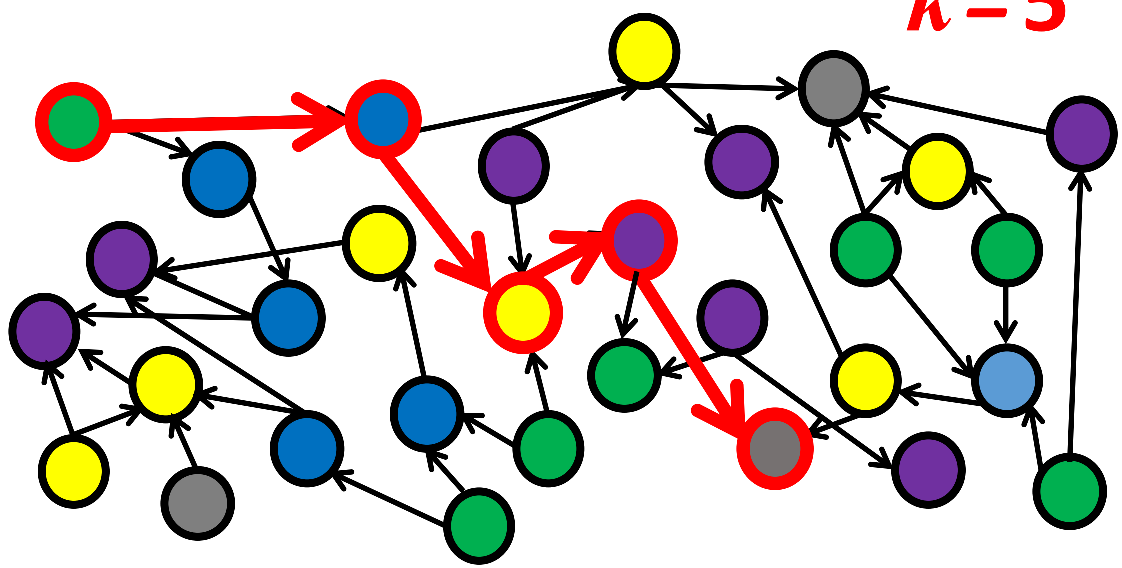
k-Path: Highlights



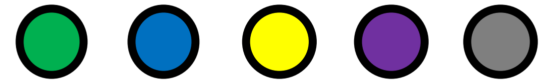
The probability of highlighting a solution:

$$1/k^k.$$

$k=5$



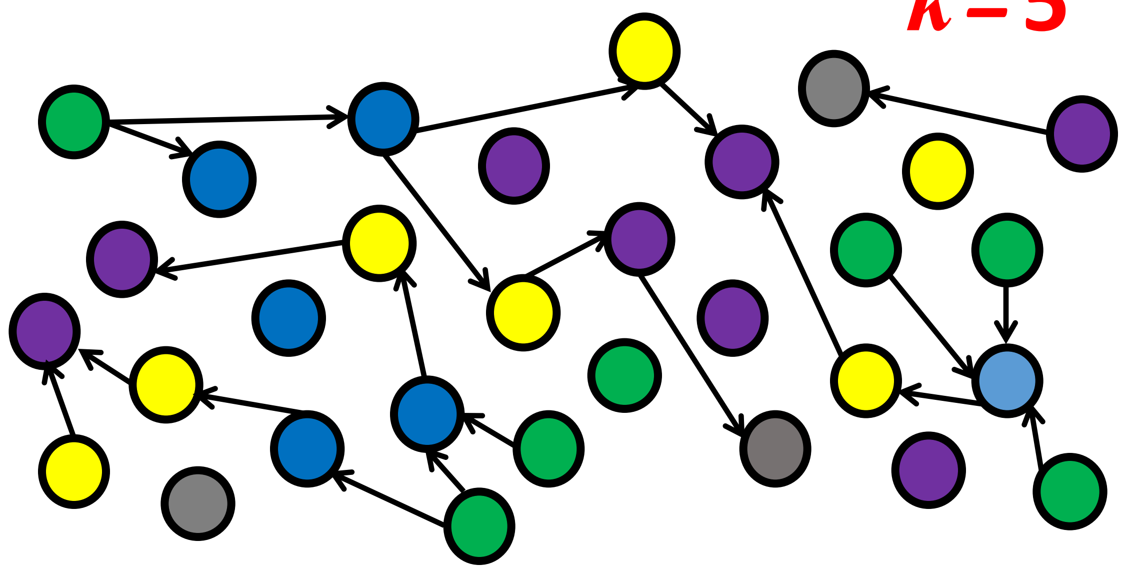
k-Path: Highlights



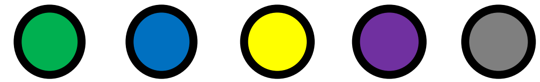
In each iteration:

- Remove irrelevant edges.
- Is there a path from a vertex colored 1 to a vertex colored k?

$k = 5$

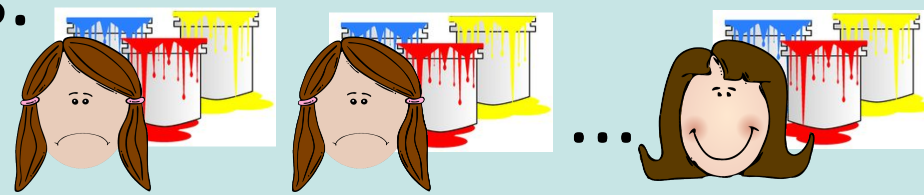


k-Path: Highlights

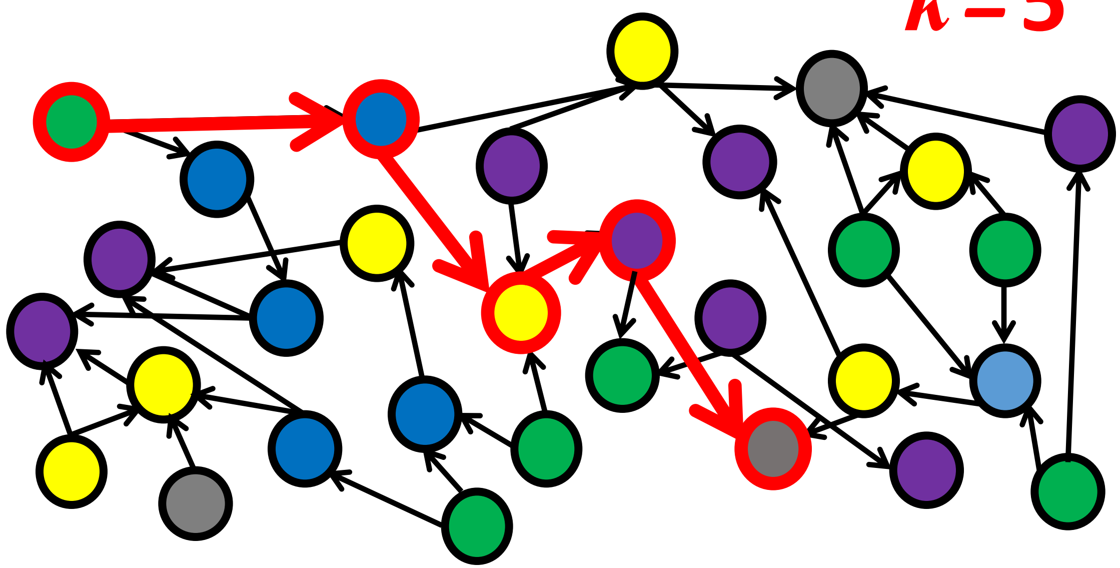


Probability of highlighting a solution: $1/k^k$.

$\rightarrow O^*(k^k)$ iterations.



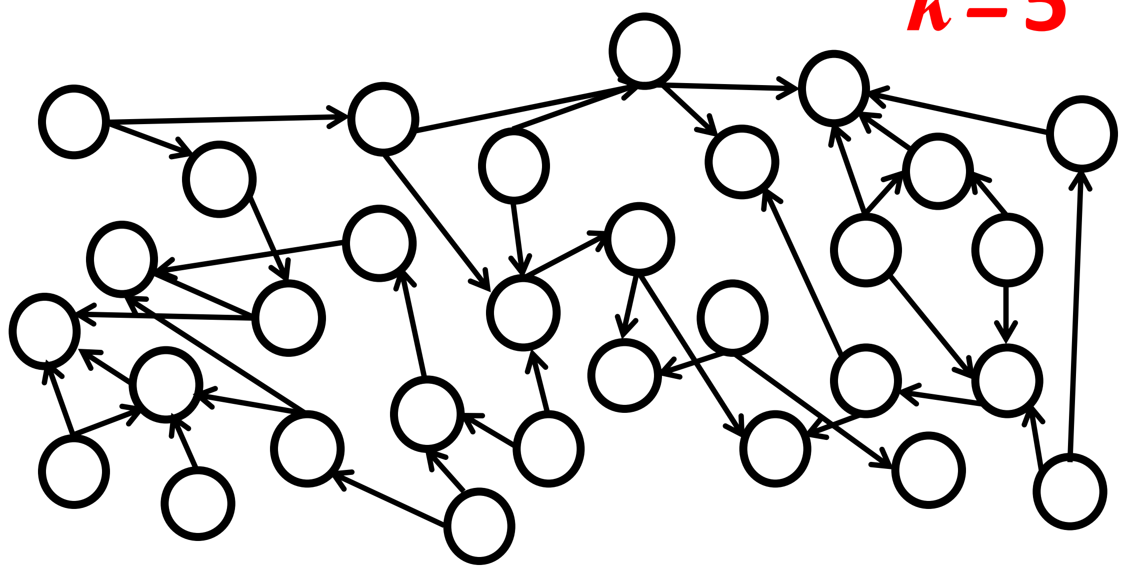
$k=5$



k-Path: Highlights

Running time: $O^*(k^k)$.

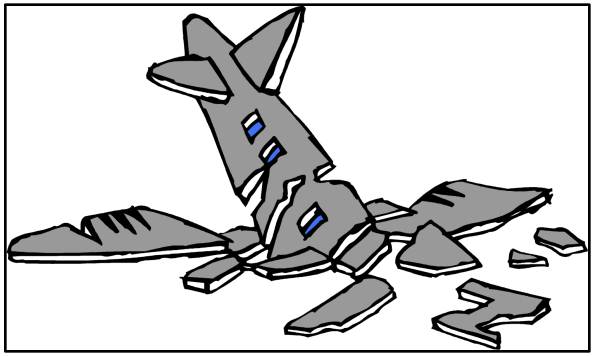
$k = 5$

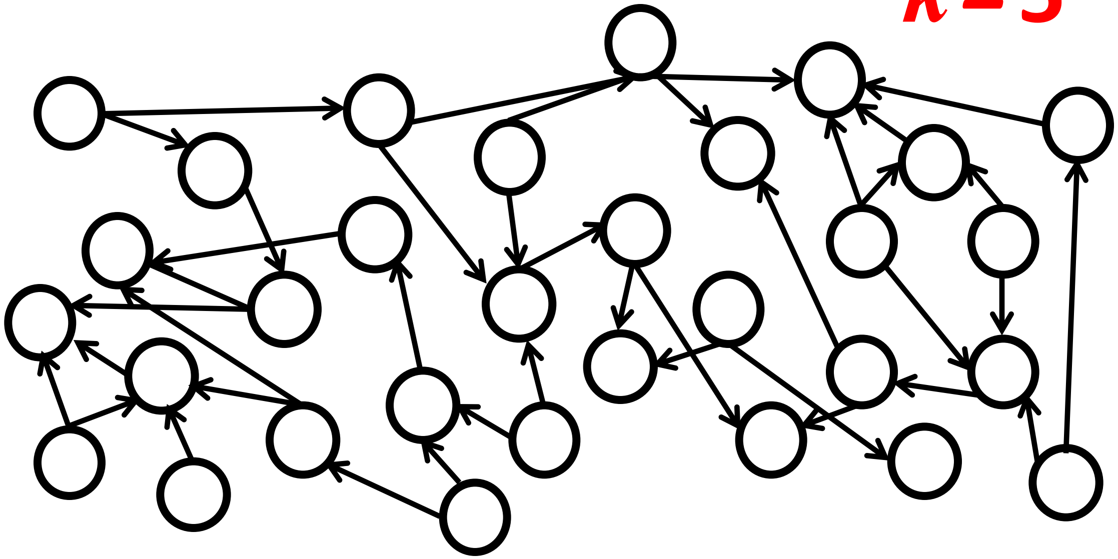


k-Path: Highlights

Running time: $O^*(k^k)$. **[Randomized.]**

$k=5$

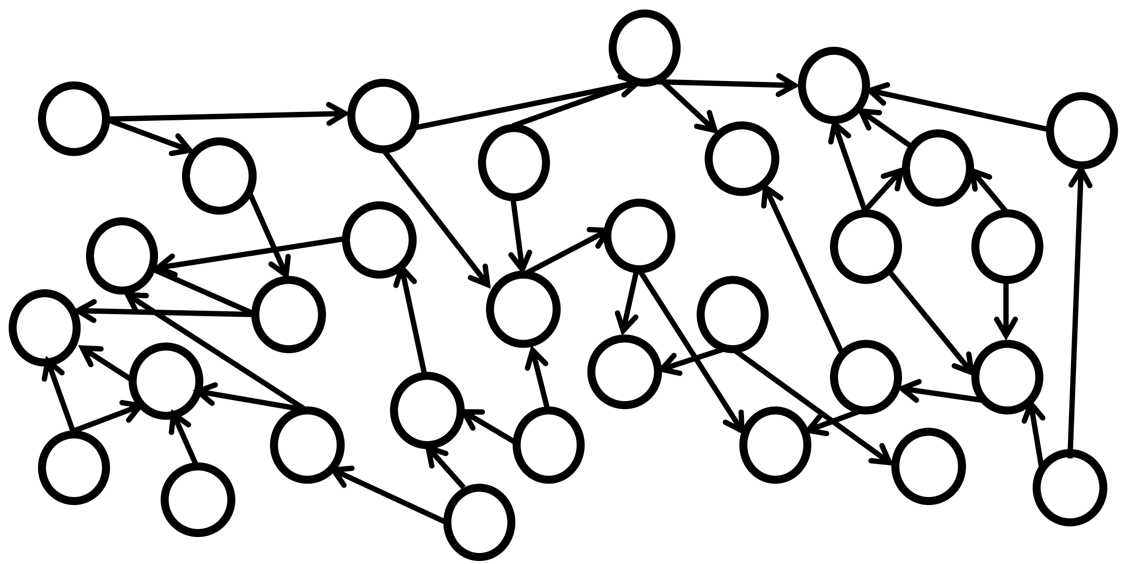




k-Path: Highlights

Derandomization:

A family F of functions f:$[n]\rightarrow[k]$ such that for all $I \subseteq [n]$ of size k and function g:$I\rightarrow[k]$, there exists $f' \in F$ that ``agrees'' with g.



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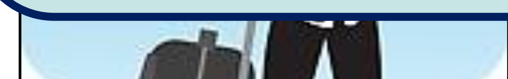
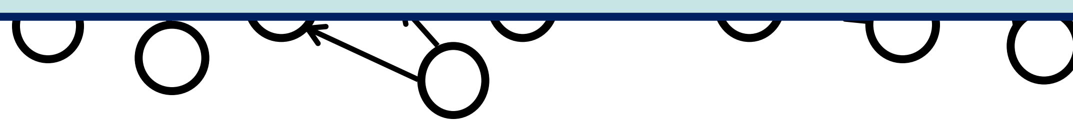
A family F of functions f:[n]⟶[k] such that for all $I\subseteq$ [n] of size k and function g:I⟶[k], there exists $f' \in F$ that ``agrees'' with g.

$|F|=O^*(k^k\cdot 2^{o(k)})$ in time $O^*(k^k\cdot 2^{o(k)})$.

Useful to know: k-wise independent sample space; (n,k)-perfect hash family.

[Alon, Babai and Itai, '86; Alon, Yuster and Zwick, '95].





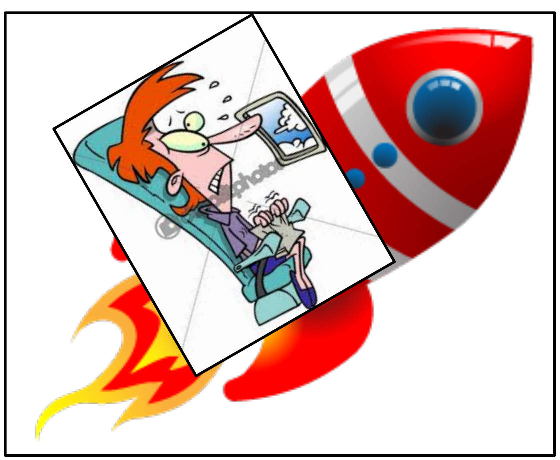
k-Path: Highlights

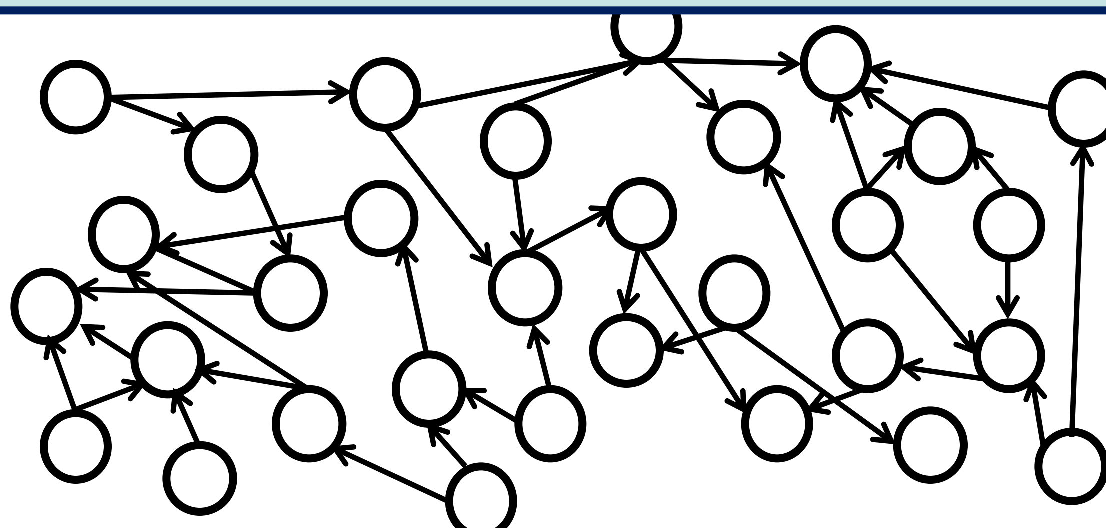
Koutis and Williams, '09:

$O^*(2^k)$. **[Randomized.]**

Björklund, Husfeldt, Kaski and Koivisto, '10:

$O^*(1.657^k)$. **[Randomized; Undirected.]**





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Polynomial identity testing.

(incl. algebraic interpretation of ideas presented in this talk.)

k-Path: Highlights

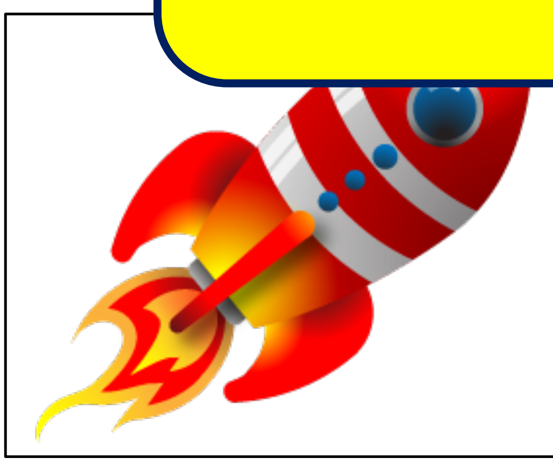
Koutis and Williams, '09:

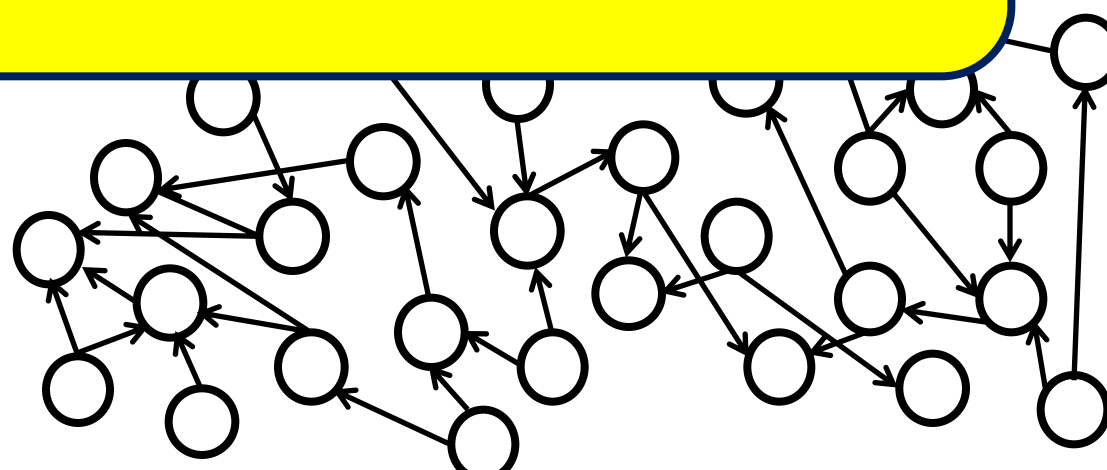
$O^*(2^k)$. [Randomized.]

Bjö ... '10:

O^* ...

Do we really need to order our colors?





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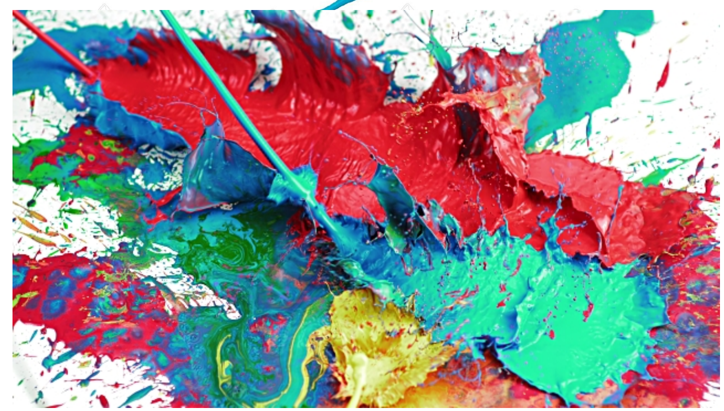
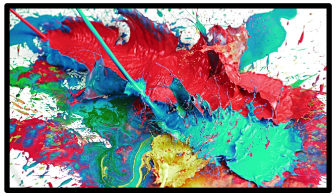
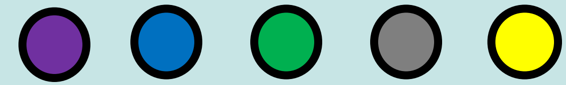


Alon, Yuster and Zwick, '95

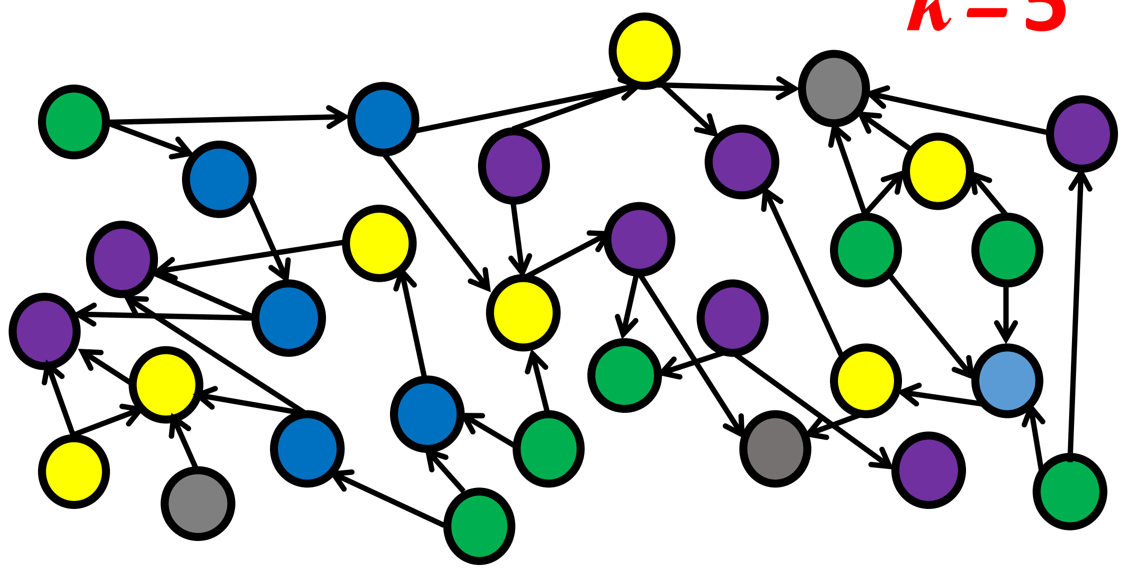


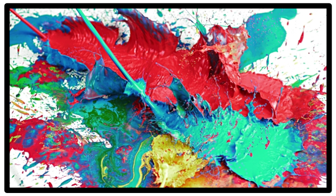
k-Path: Color Coding

Again: Color-set: $\{1,2,\ldots,k\}$. 

To each vertex, randomly assign a color.

$k=5$





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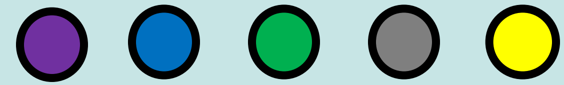
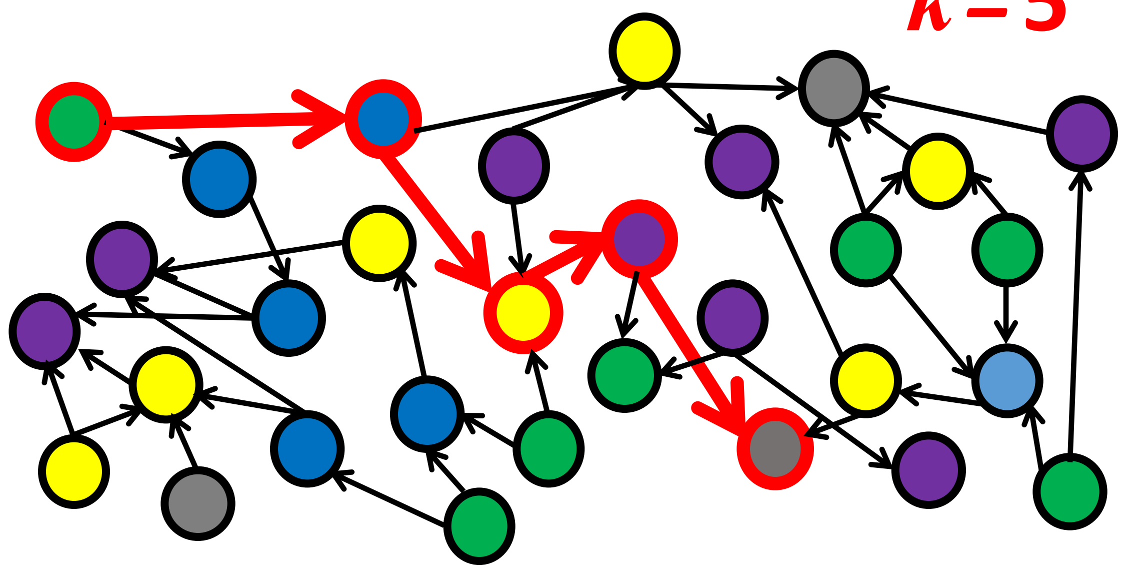
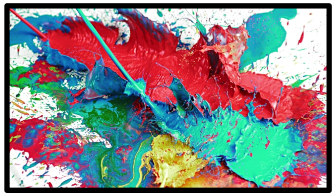
Again: Color-set: $\{1,2,\ldots,k\}$. 

To each vertex, randomly assign a color.

Easier request: Colorful solution.

$k=5$



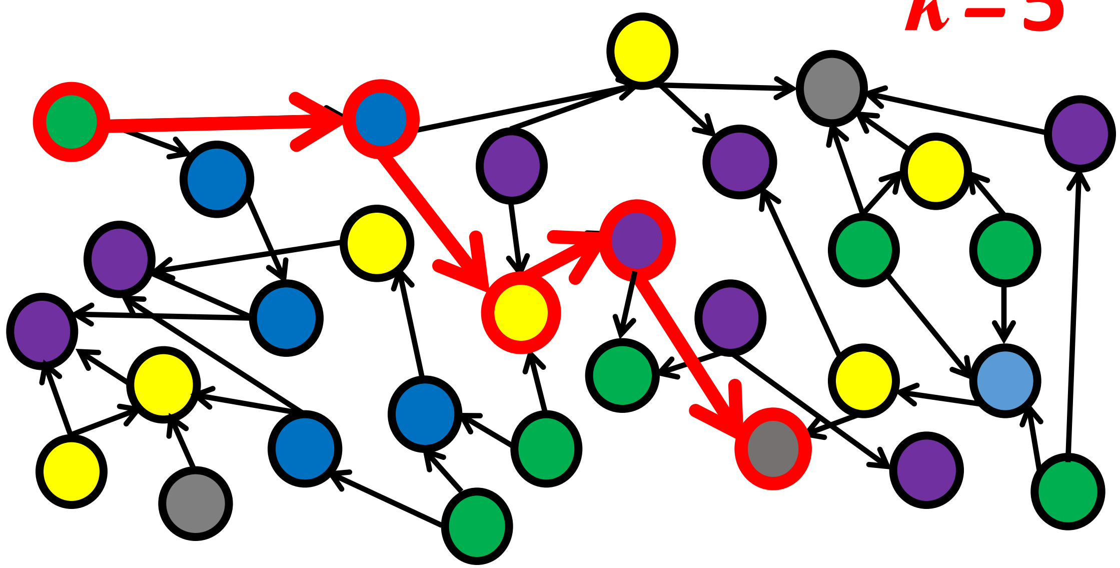


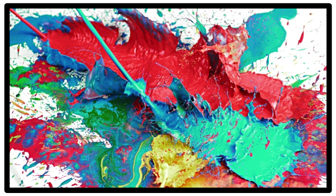
k-Path: Color Coding

The probability of coloring a solution correctly:

$$k!/k^k \approx 1/e^k.$$

$k=5$





k-Path: Color Coding

In each iteration:

- $\mathrm{M}[v, S]$: Is there an S-colorful path that ends at v?

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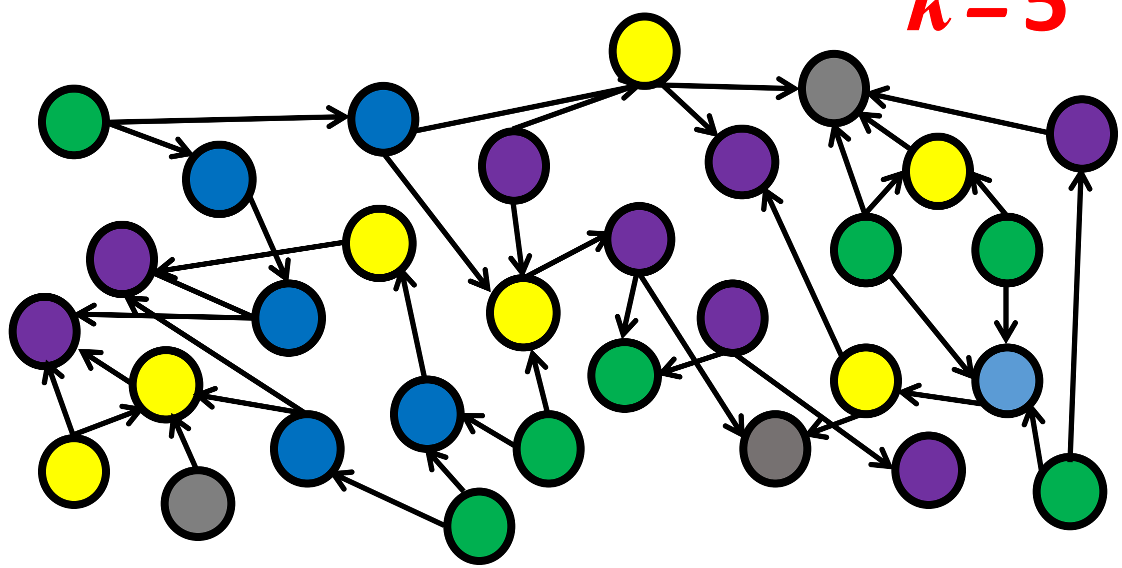
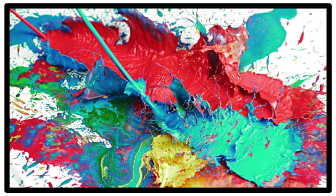


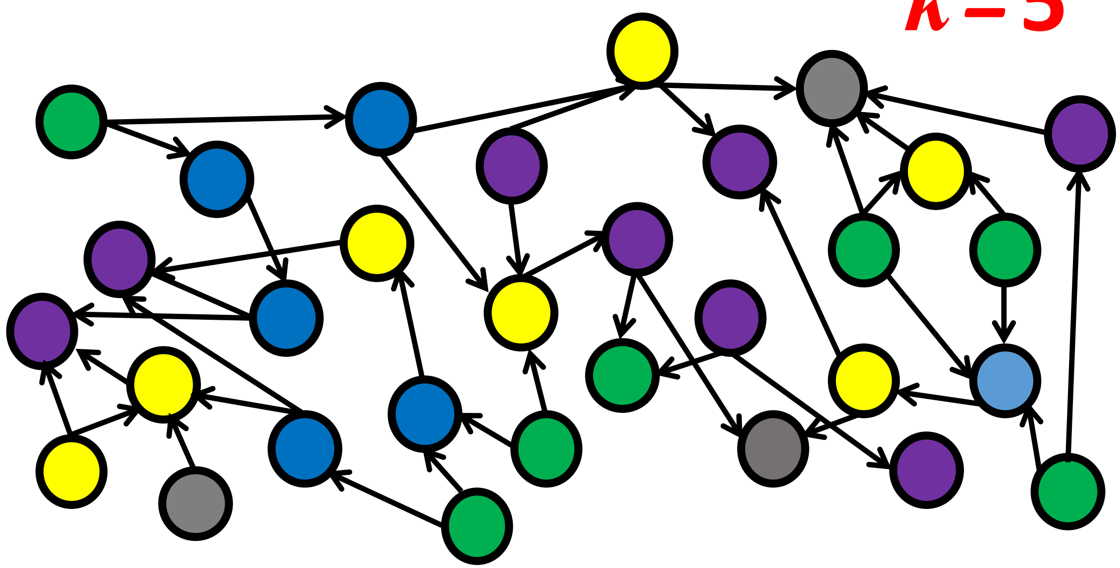


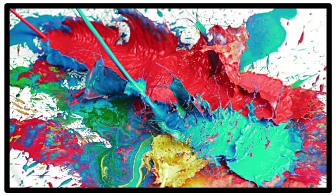
k-Path: Color Coding

In each iteration:

- $M[v,S]$: Is there an S-colorful path that ends at v?
- $M[v,S] = \vee_{(u,v)\in E} M[u, S \setminus \{color(v)\}]$

$k = 5$





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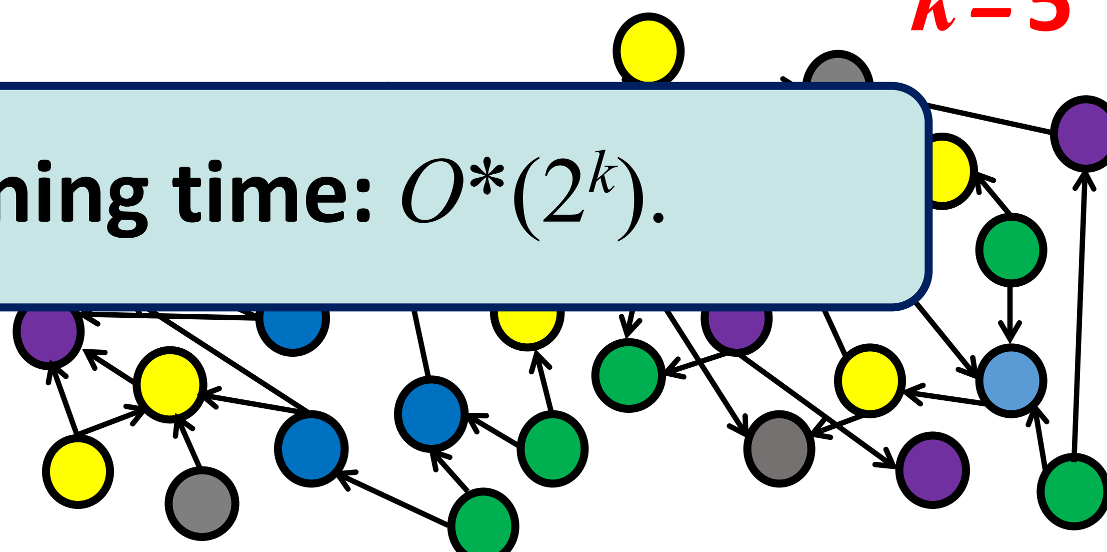
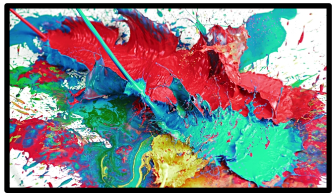


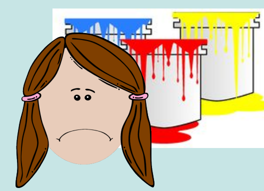
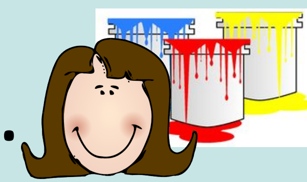
Running time: $O^*(2^k)$.



k-Path: Color Coding

Probability of coloring a sol correctly: $1/e^k$.

→ $O^*(e^k)$ iterations.

  … 

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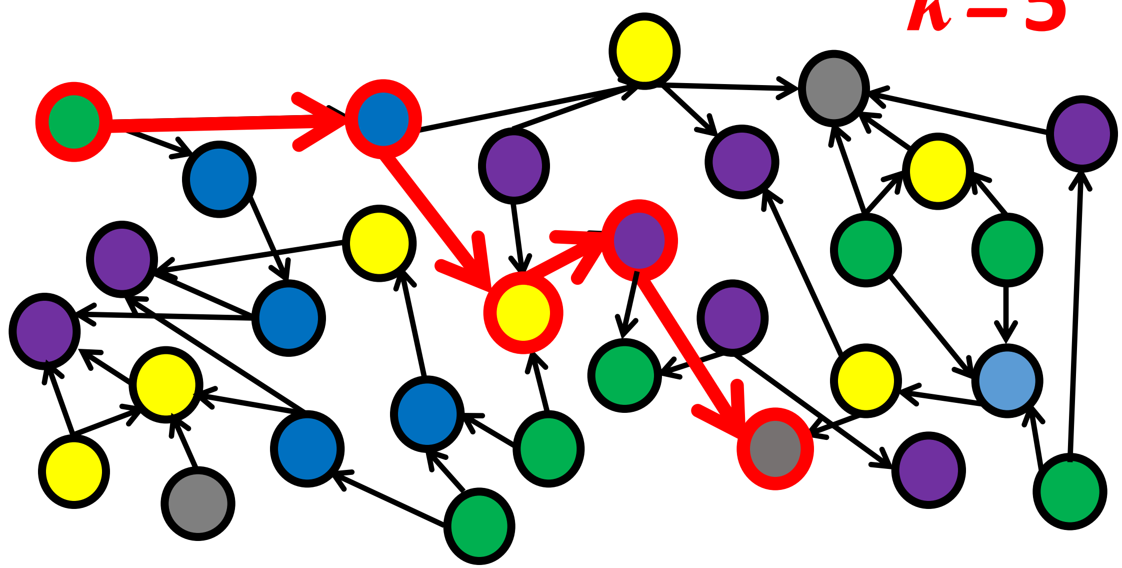
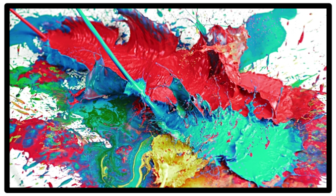
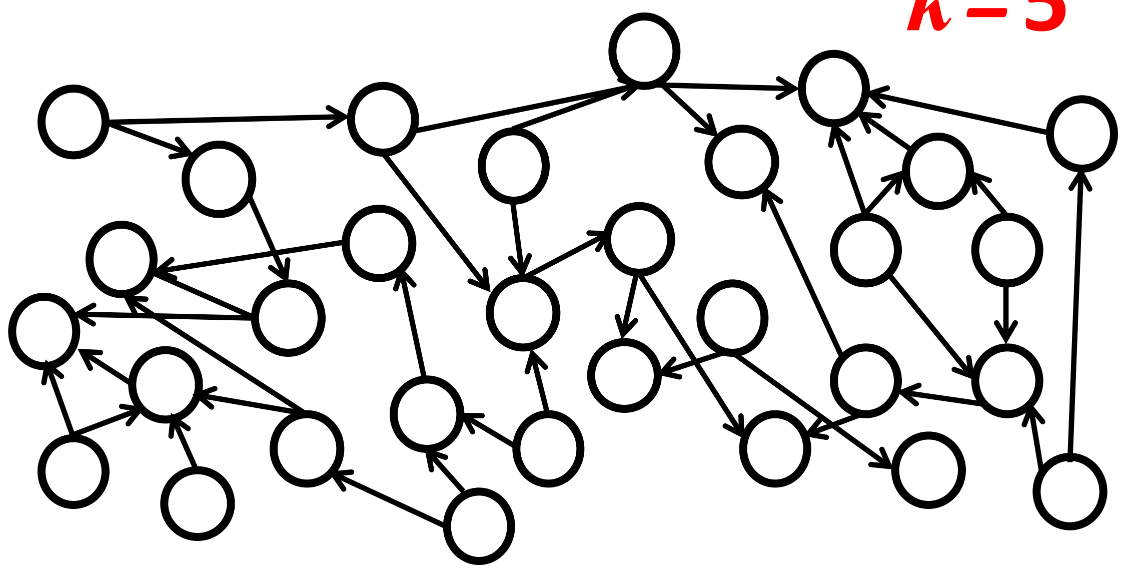


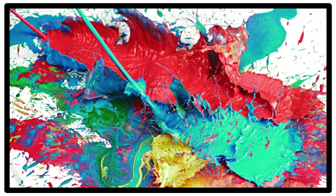


k-Path: Color Coding

Running time: $O^*((2e)^k)$.

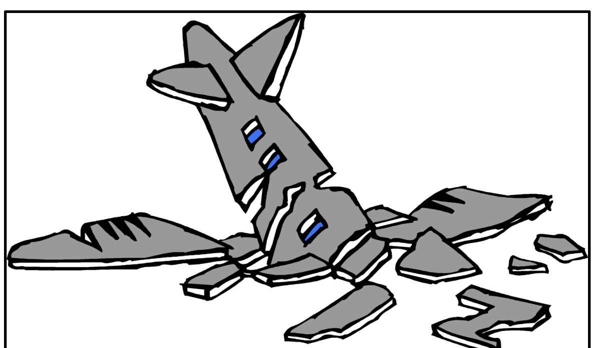
$k = 5$



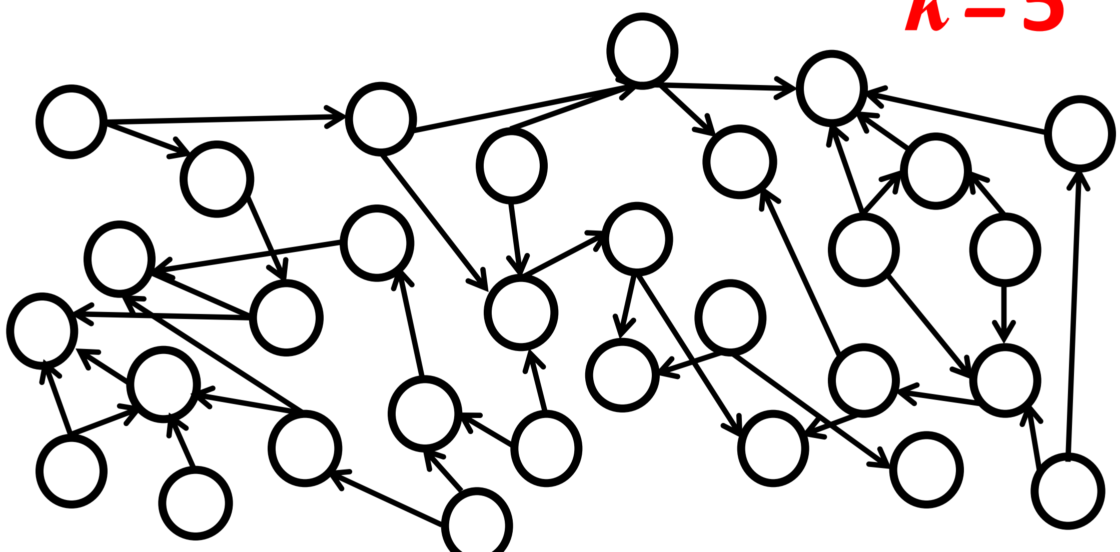


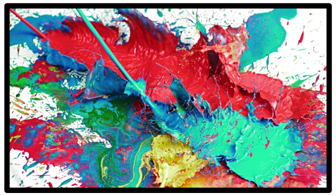
k-Path: Color Coding

Running time: $O^*((2e)^k)$. **[Randomized.]**



***k*=5**

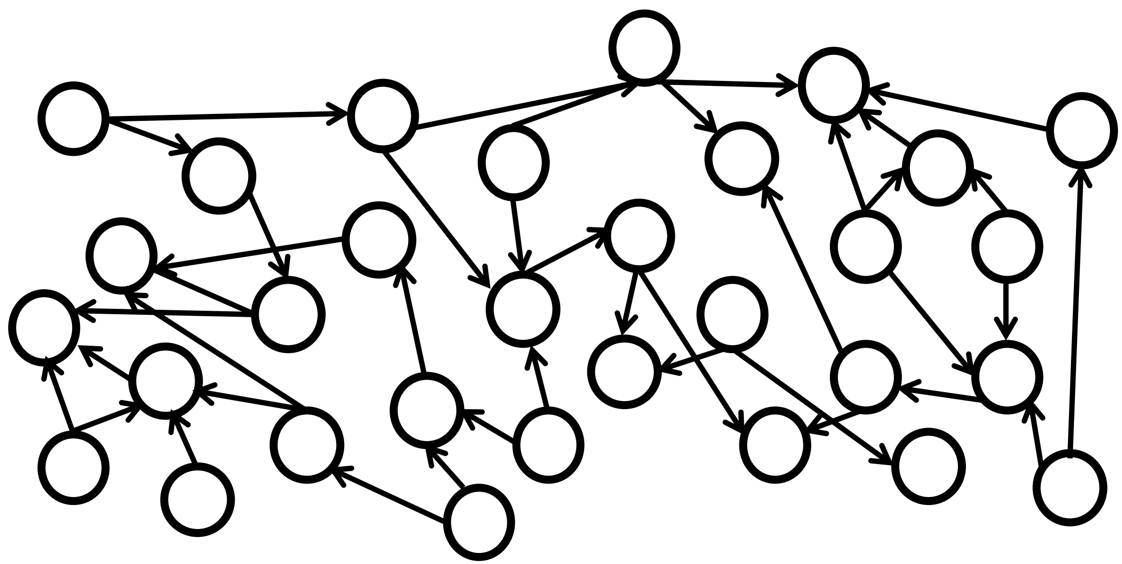


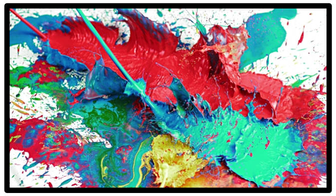


k-Path: Color Coding

Derandomization:

A family F of functions f:$[n]\rightarrow[k]$ such that for all $I \subseteq [n]$ of size k, there exists $f' \in F$ such that $f'|_I$ is an injective function.





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Derandomization:

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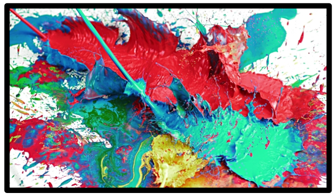
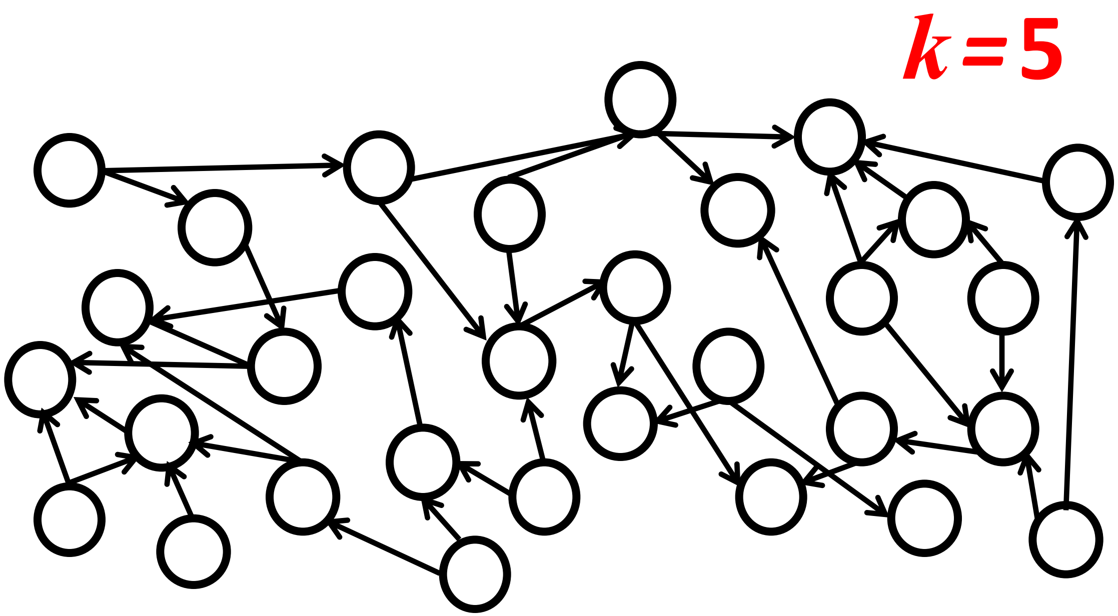
$|F|=O^*(e^{k+o(k)})$ **in time** $O^*(e^{k+o(k)})$.

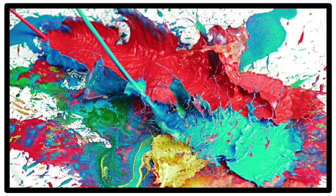
Useful to know: (n,k,l)-splitter; (n,k)-perfect hash family; δ-balanced (n,k)-perfect hash family. [Naor, Schulman and Srinivasan, '95; Alon, Yuster and Zwick, '95; Alon and Gutner, '10].



k-Path: Color Coding

Running time: $O^*((2e)^{k+o(k)})$. **[Deterministic.]**





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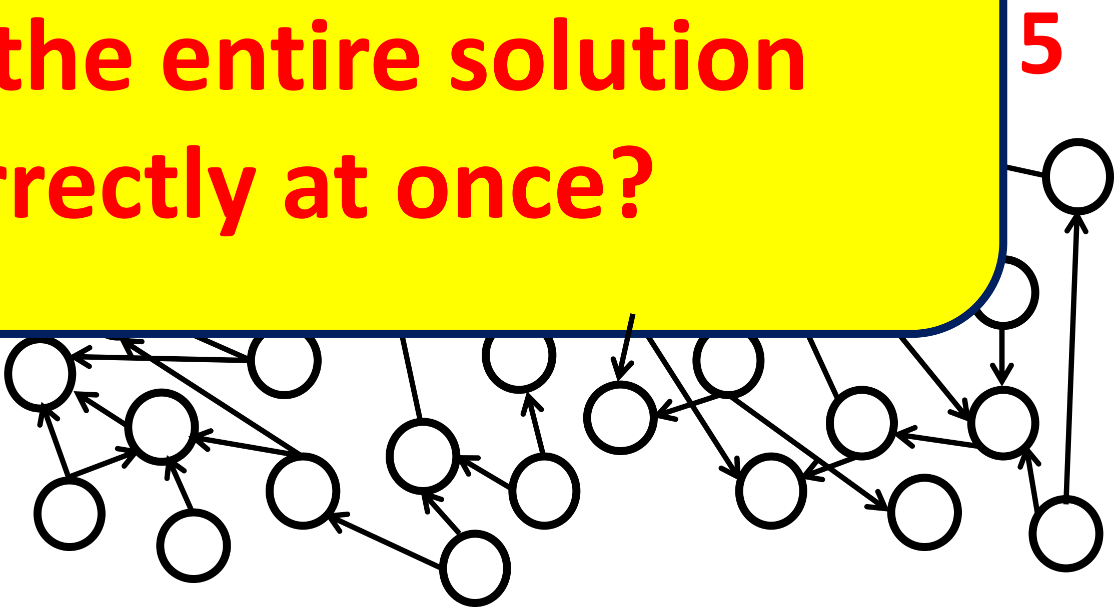


Do we really need to color the entire solution correctly at once?

(Art?) Tutorial

1. Brute-Force
2. Highlights
3. Color Coding
4. Divide-and-Color
5. Representative Sets
6. Mixing

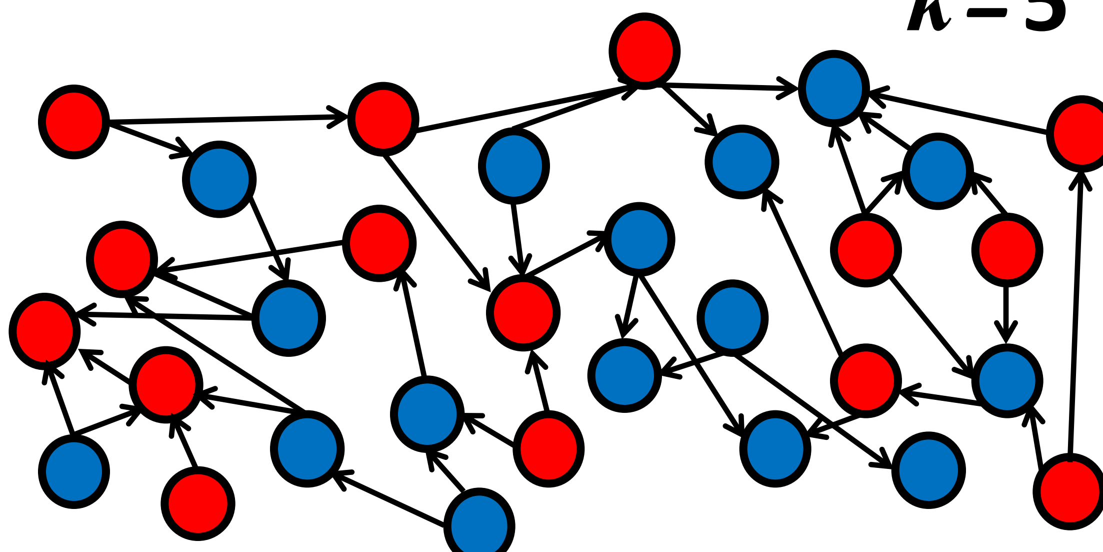
Chen, Kneis, Lu, Mölle, Richter, Rossmanith, Sze and Zhang, '09

k-Path: Divide-and-Color

Color-set: $\{\mathbf{R},\mathbf{B}\}$.

To each vertex, randomly assign a color.

$k=5$

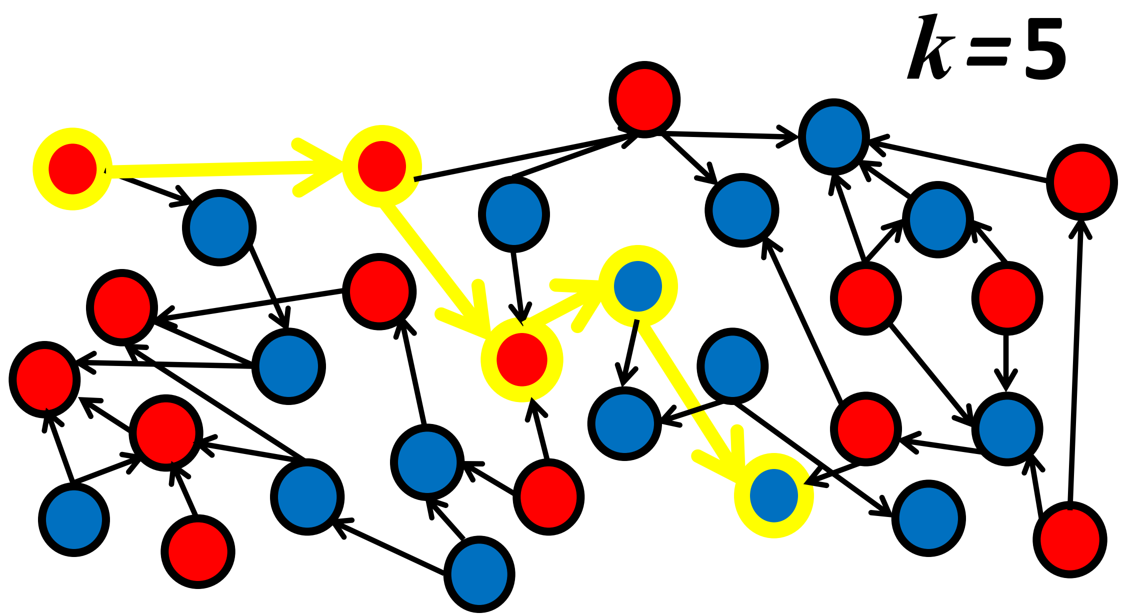


k-Path: Divide-and-Color

Color-set: $\{\mathbf{R},\mathbf{B}\}$.

To each vertex, randomly assign a color.

Easier request: Divide a solution $(\lceil k/2 \rceil, \lfloor k/2 \rfloor)$.



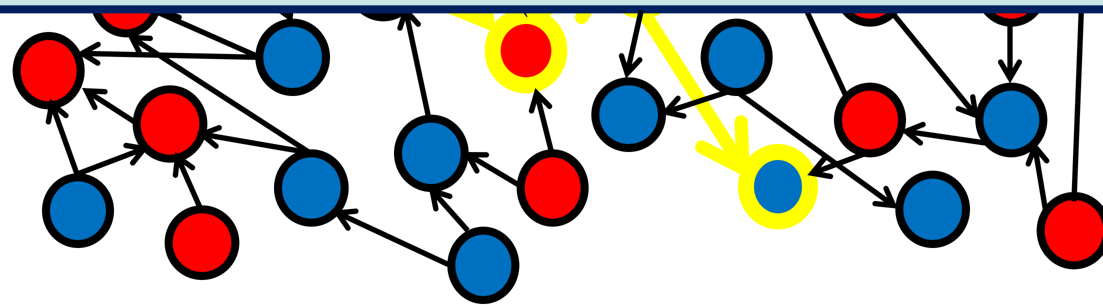
k-Path: Divide-and-Color

Color-set: $\{\mathbf{R},\mathbf{B}\}$.

To each vertex, randomly assign a color.

Easier request: Divide a solution $(\lceil k/2\rceil,\lfloor k/2\rfloor)$.

Multiple paths: For all $u,v \in V$, does there exist a (u,v)-path?

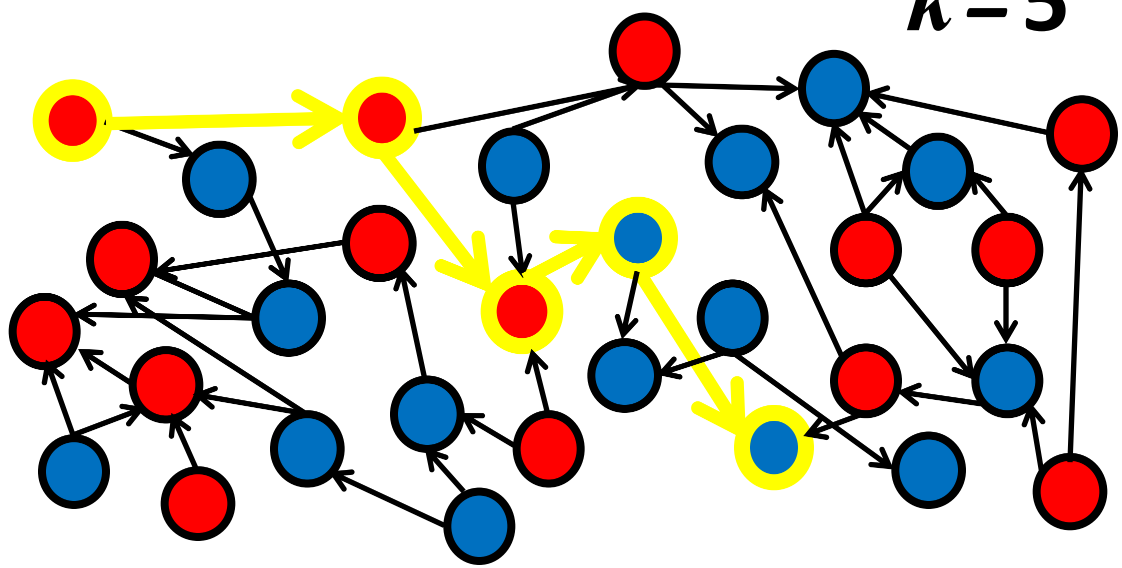


k-Path: Divide-and-Color

The probability of dividing a solution:

$1/2^k$.

$k=5$

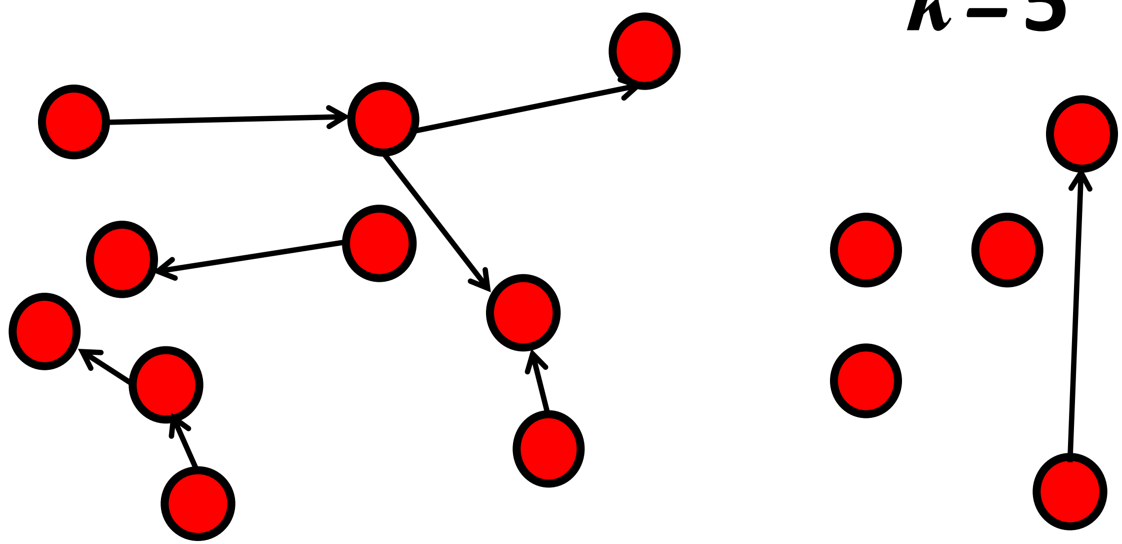


k-Path: Divide-and-Color

In each iteration:

- **Red graph**: solve $[k/2]$-Path (recursive call).

$k = 5$

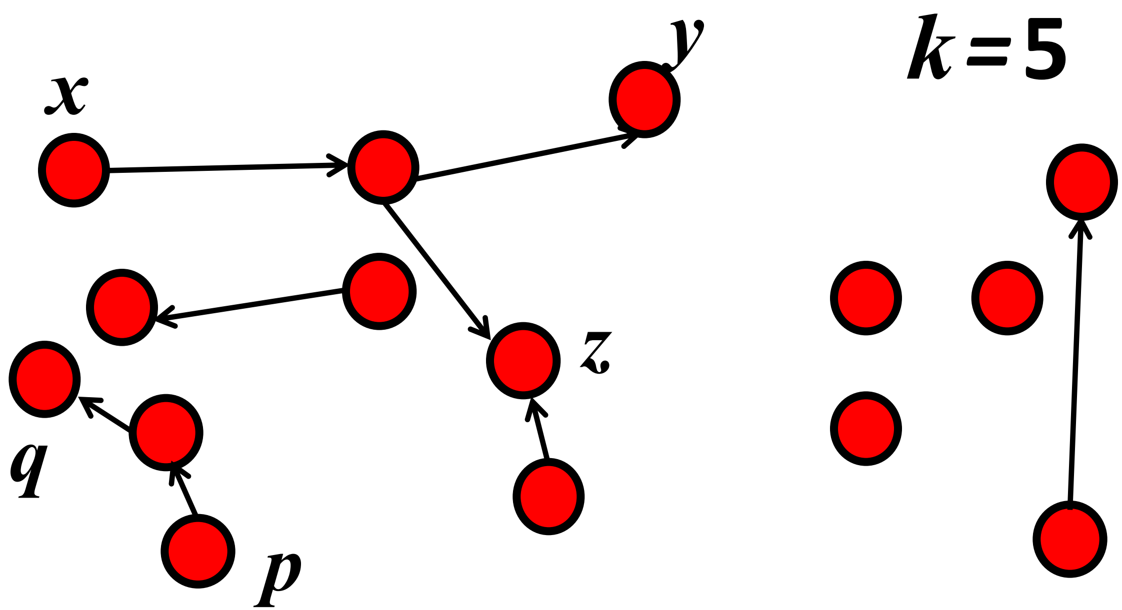


k-Path: Divide-and-Color

In each iteration:

- **Red graph**: solve $[k/2]$-Path (recursive call).

$(x,y); (x,z); (p,q).$

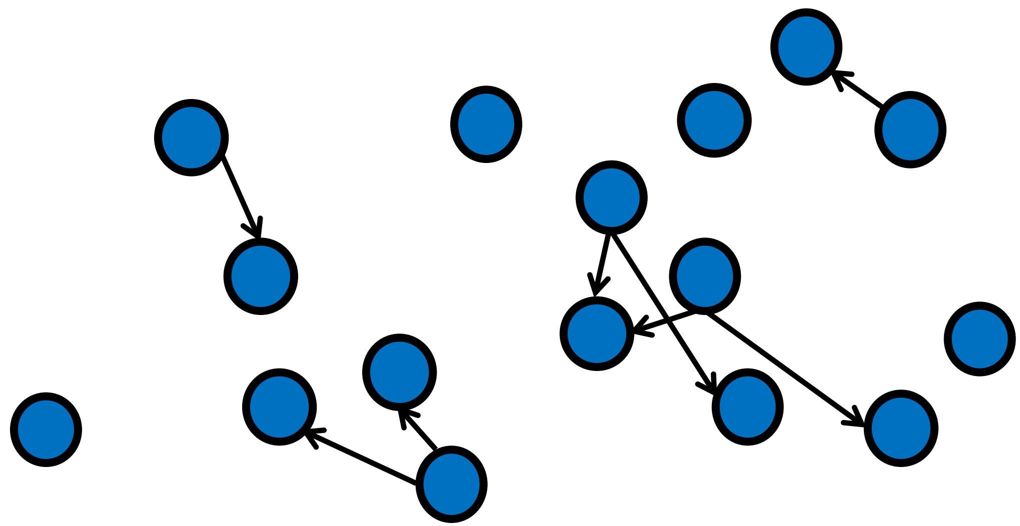


k-Path: Divide-and-Color

In each iteration:

- $(x,y);(x,z);(p,q)$.
- **Blue graph**: solve $\lfloor k/2 \rfloor$-Path (recursive call).

$k = 5$



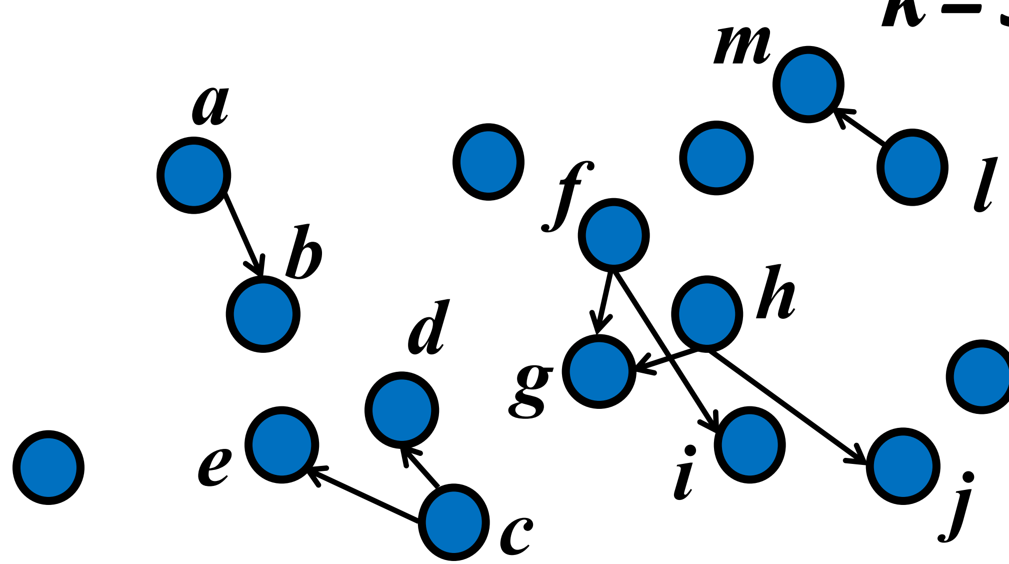
k-Path: Divide-and-Color

In each iteration:

- $(x,y);(x,z);(p,q)$.
- **Blue graph**: solve $\lfloor k/2 \rfloor$-Path (recursive call).

$(a,b);(c,d);(c,e);(f,g);(f,i);(h,g);(h,j);(l,m)$.

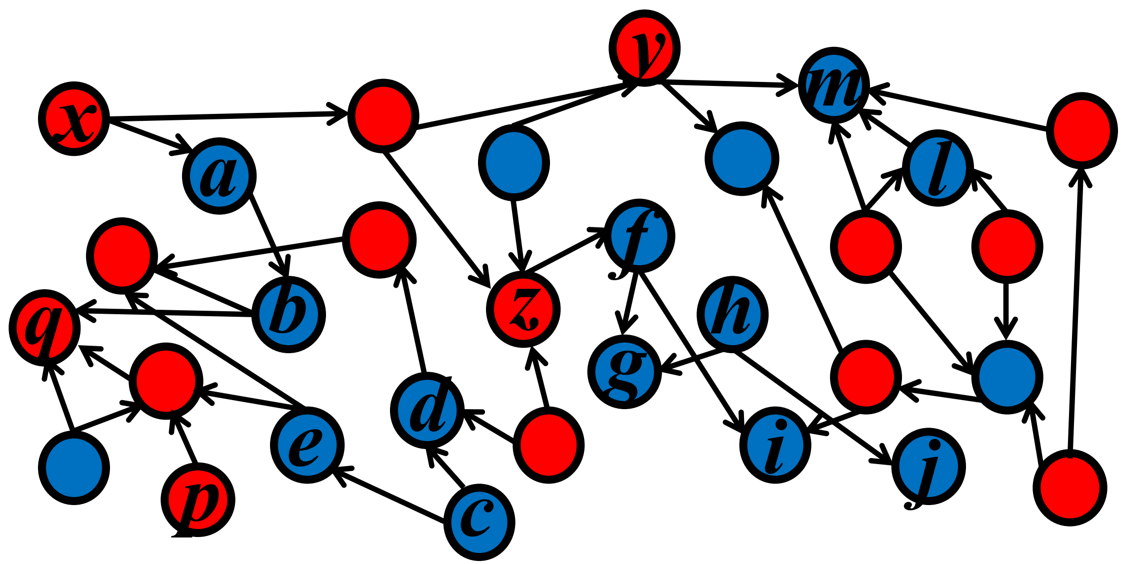
$k = 5$



k-Path: Divide-and-Color

In each iteration:

- $(x,y);(x,z);(p,q)$.
- $(a,b);(c,d);(c,e);(f,g);(f,i);(h,g);(h,j);(l,m)$.
- Glue: $(\alpha,\beta),(\gamma,\delta)$ where $(\beta,\gamma) \in E$.



k-Path: Divide-and-Color

In each iteration: (x,g)

- $(x,y);(x,z);(p,q)$.
- $(a,b);(c,d);(c,e);(f,g);(f,i);(h,g);(h,j);(l,m)$.
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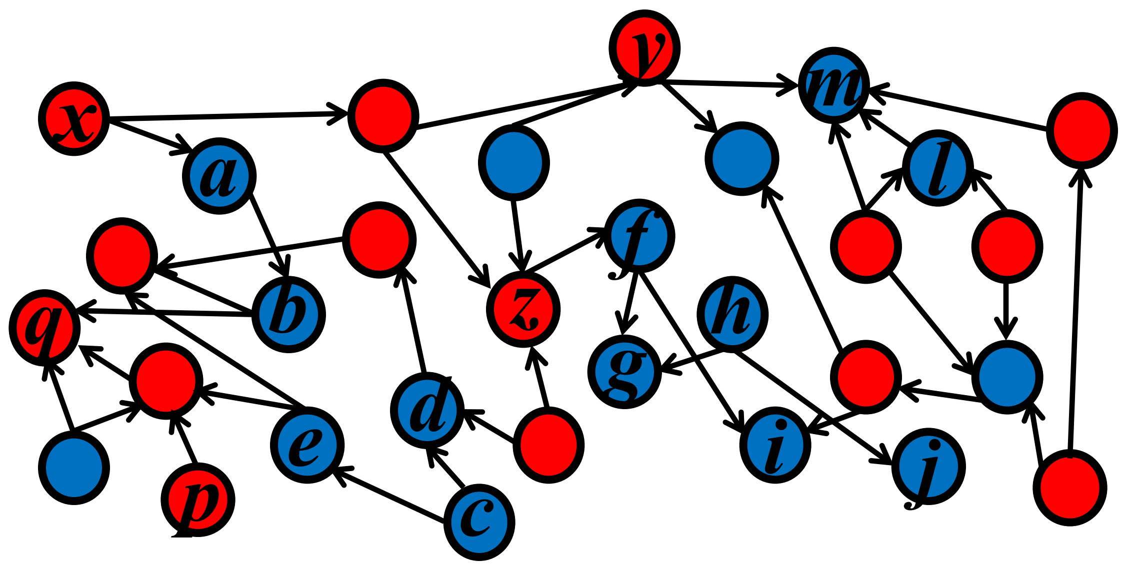
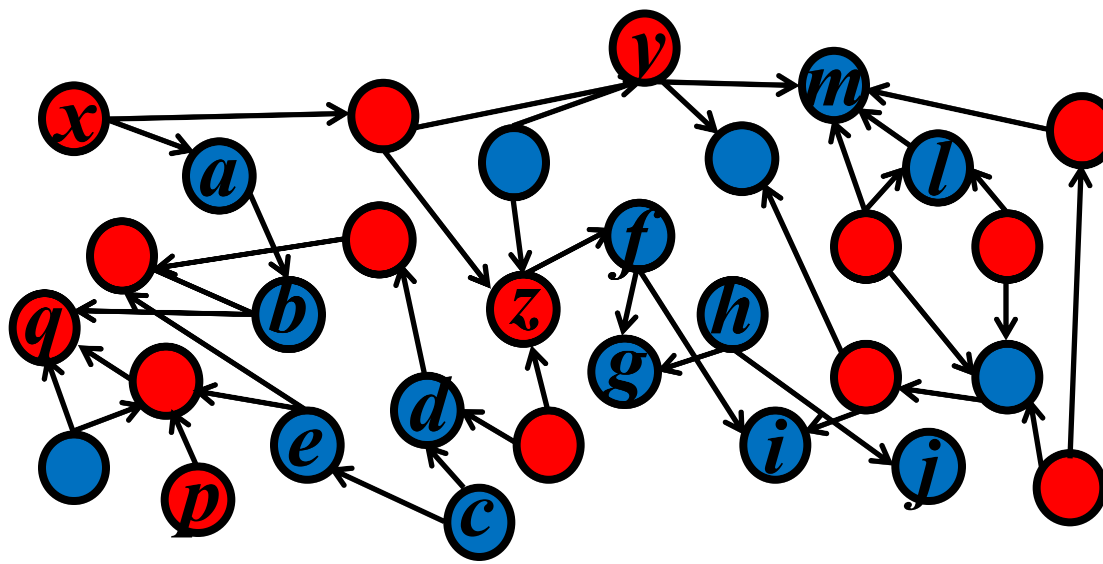


k-Path: Divide-and-Color

In each iteration: $(x,g);(x,i)$

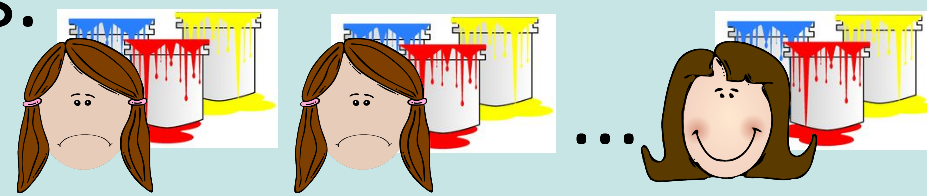
- $(x,y);(x,\underline{z});(p,q)$.
- $(a,b);(c,d);(c,e);(f,g);(f,i);(h,g);(h,j);(l,m)$.
- Glue: $(\alpha,\beta),(\gamma,\delta)$ where $(\beta,\gamma)\in E$.



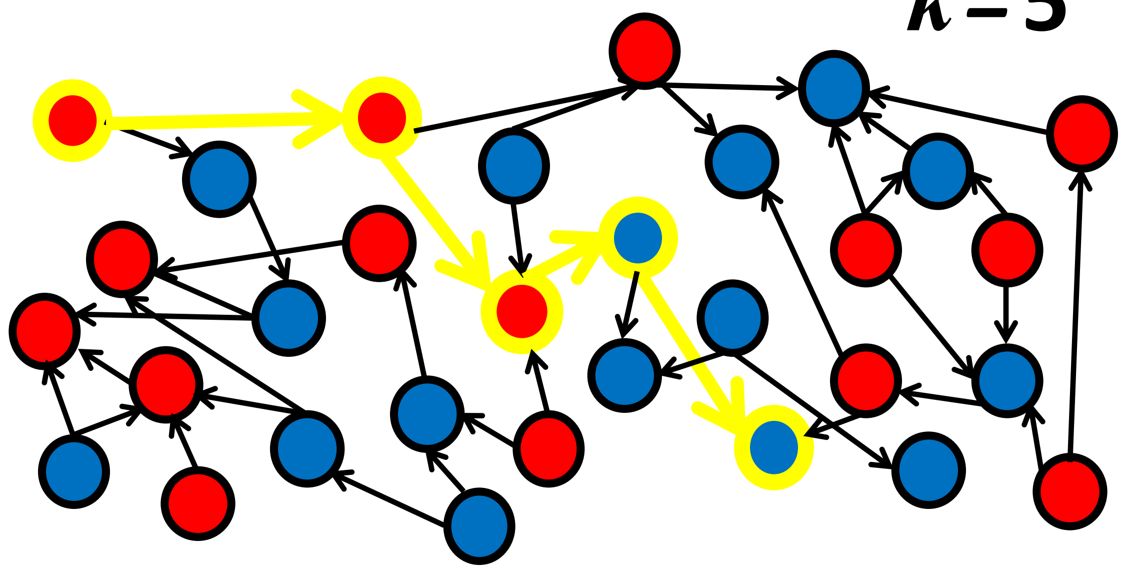
k-Path: Divide-and-Color

Probability of coloring a sol correctly: $1/2^k$.

$\rightarrow O^*(2^k)$ iterations.

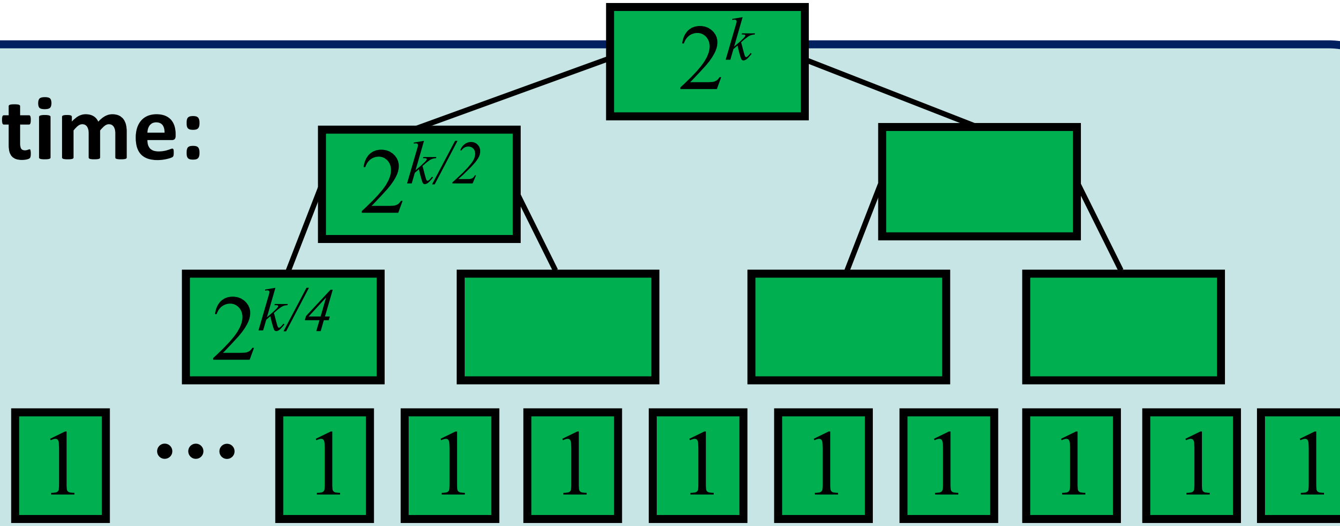


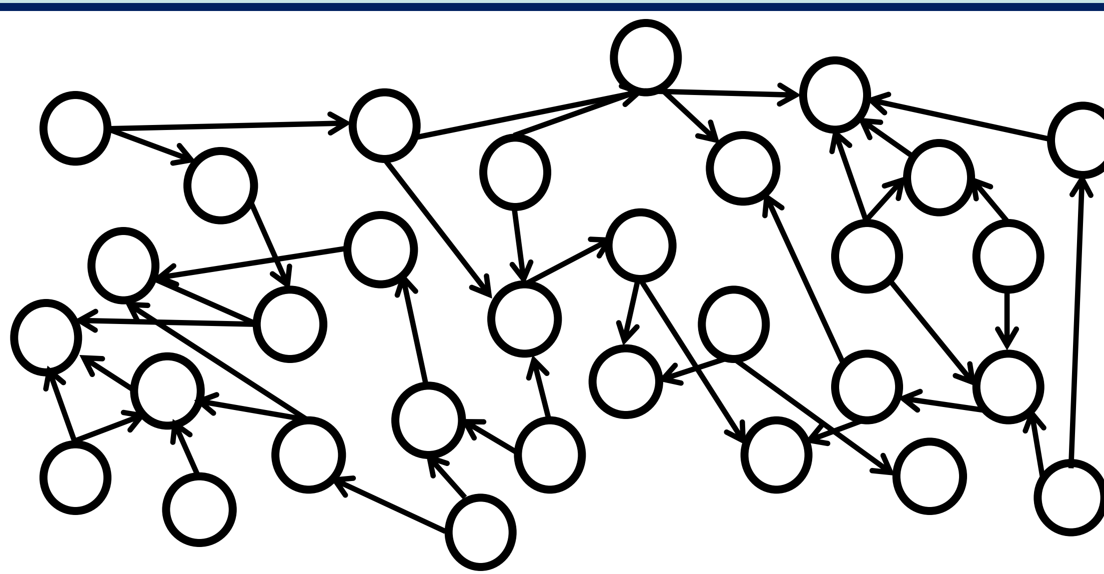
$k = 5$



k-Path: Divide-and-Color

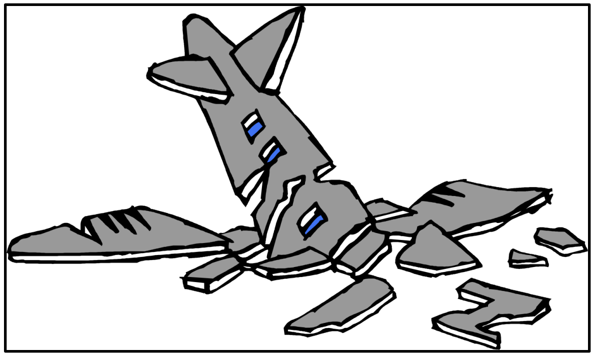
Running time:
$O^*(4^k)$.



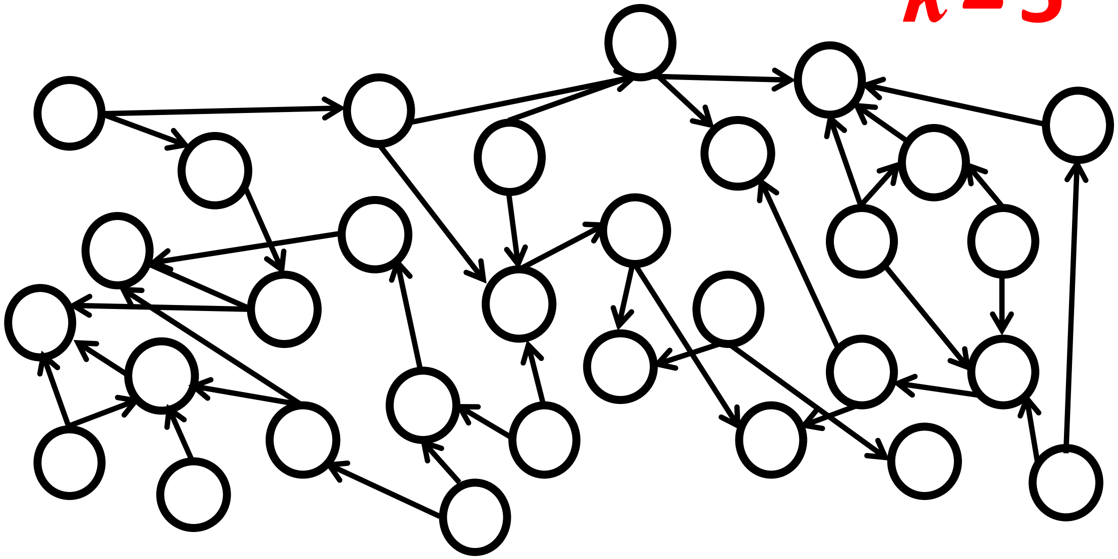


k-Path: Divide-and-Color

Running time: $O^*(4^k)$. **[Randomized.]**



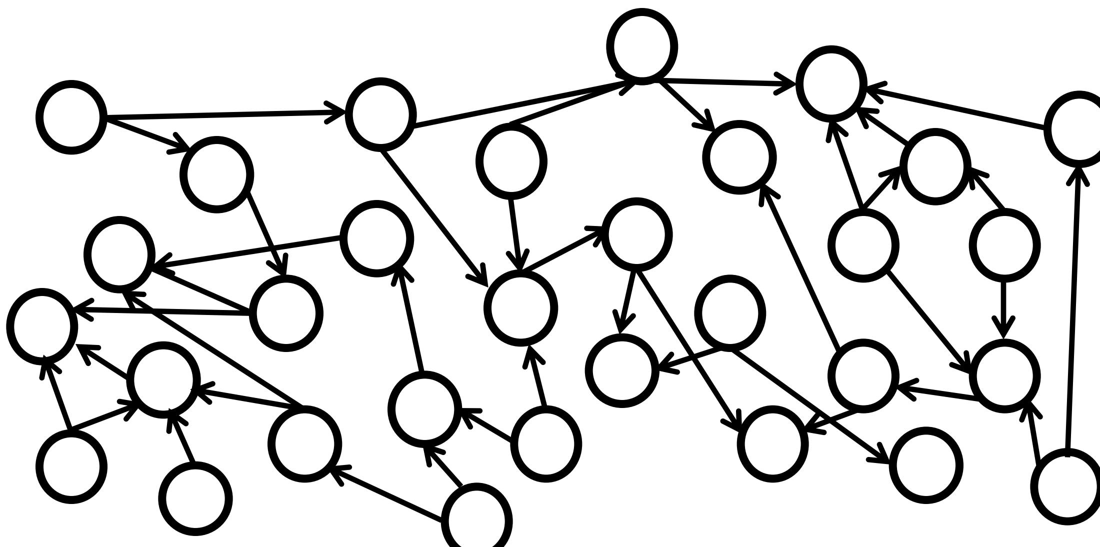
$k=5$



k-Path: Divide-and-Color

Derandomization:

A family F of functions $f : [n] \rightarrow [2]$ such that for all $I \subseteq [n]$ of size k and function $g : I \rightarrow [2]$, there exists $f' \in F$ that ``agrees'' with g.



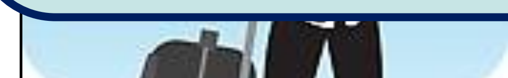
k-Path: Divide-and-Color

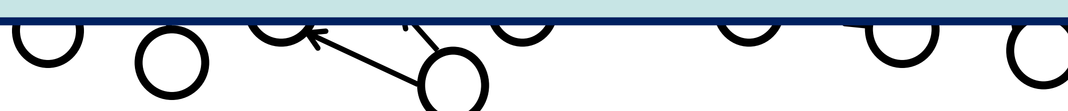
Derandomization:

A family F of functions f:$[n]\rightarrow[2]$ such that for all $I\subseteq[n]$ of size k and function g: $I\rightarrow[2]$, there exists $f'\in F$ that ``agrees'' with g.

$|F|=O^*(2^{k+o(k)})$ in time $O^*(2^{k+o(k)})$.

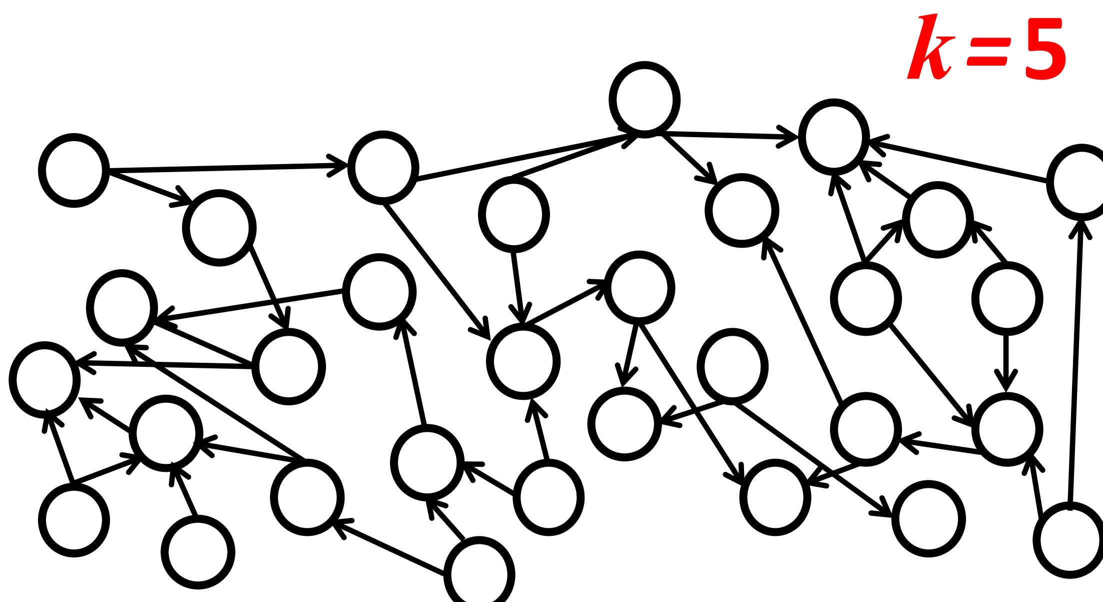
Useful to know: (n,k)-universal set; (n,k,p)-universal set. [Naor, Schulman and Srinivasan, '95; Fomin, Lokshtanov, Panolan and Saurabh, '16].





k-Path: Divide-and-Color

Running time: $O^*(4^{k+o(k)})$. **[Deterministic.]**

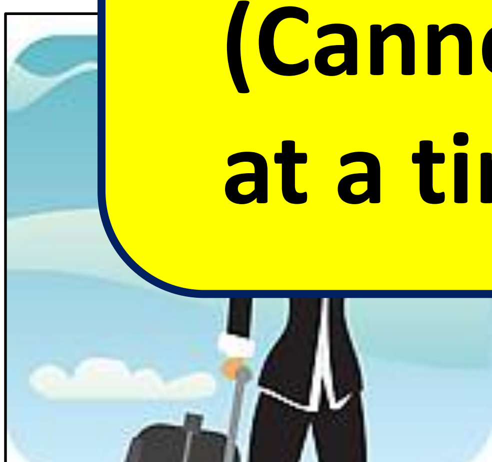


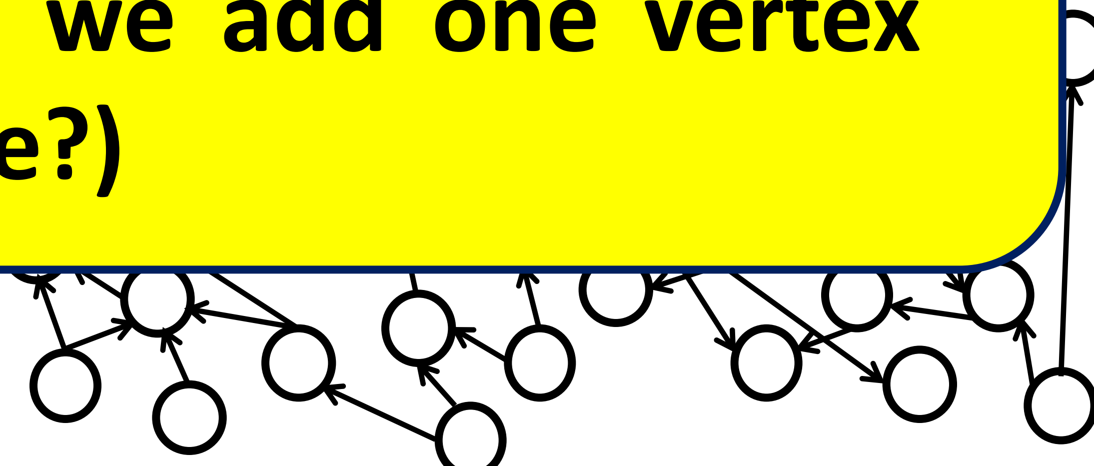
k-Path: Divide-and-Color

Running time: $O^*(4^{k+o(k)})$. **[Deterministic.]**

Do we really need to divide the entire solution at once?

(Cannot we add one vertex at a time?)





(Art?) Tutorial

1. Brute-Force
2. Highlights
3. Color Coding
4. Divide-and-Color
   - Extra Example
5. Representative Sets
6. Mixing

Zehavi, '16

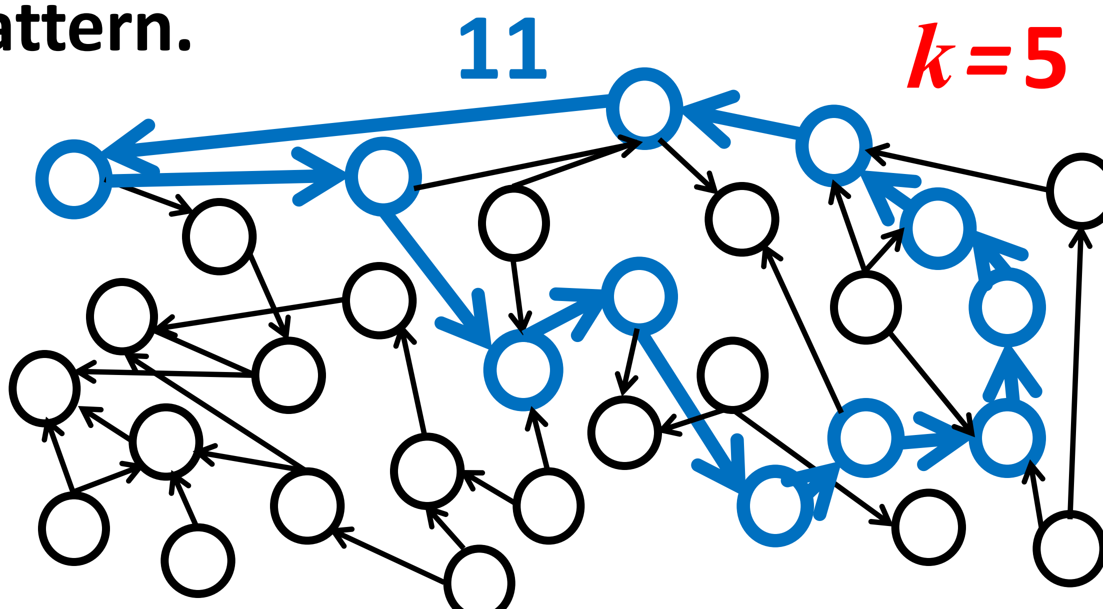
Extra Example: Long Cycle

Long Cycle.

Input: Directed graph G; parameter k.

Question: Does G have a cycle on **at least** k vertices?

Finding a large pattern.



Extra Example: Long Cycle

Long Cycle.

Input: Directed graph G; parameter k.

Question: Does G have a cycle on **at least** k vertices?

Running time:

Deterministic. $O^*(\max\{\mathbf{P(2}\boldsymbol{k}\mathbf{)}, 4^{k+o(k)}\})$

Randomized. $O^*(4^k)$. **Zehavi, '16**

solve $2k$-Path

Extra Example: Long Cycle

Long Cycle.

Input: Directed graph G; parameter k.

Question: Does G have a cycle on **at least** k vertices?

Running time:

Deterministic. $O^*(\max\{\mathbf{P(2k)},\ 4^{k+o(k)}\})$

Randomized. $O^*(4^k)$. **Zehavi, '16**

Step 1.

Determine whether G has a t-cycle for $t \in \{k,\ldots,2k\}$.

Extra Example: Long Cycle

Step 2 (multiple times).

i. To each vertex, assign a color (**R**/**B**).

Extra Example: Long Cycle

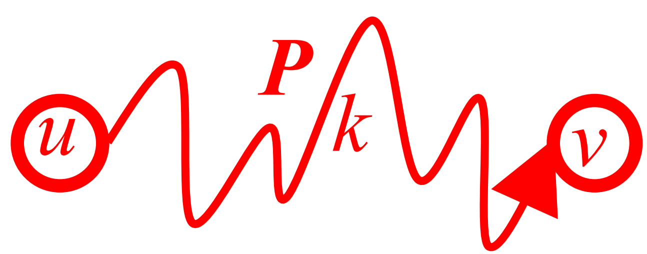
Step 2 (multiple times).

i. To each vertex, assign a color (**R**/**B**).

ii. For all $u,v \in$ **R**:

a) $\boldsymbol{P}$ - shortest (u,v)-path in $G[$**R**$]$ (BFS).

If $|V(\boldsymbol{P})|\neq k$: Next iteration.



Extra Example: Long Cycle

Step 2 (multiple times).

i. To each vertex, assign a color (**R**/**B**).

ii. For all $u,v \in$ **R**:

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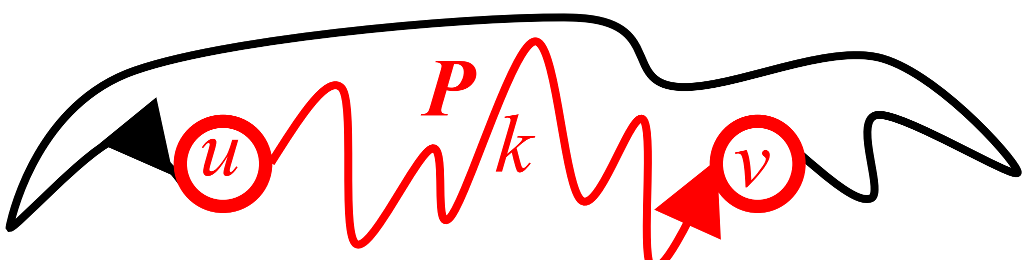
If $|V(\boldsymbol{P})| \neq k$: Next iteration.

b) If there is a (v,u)-path in $G \backslash (V(\boldsymbol{P}) \backslash \{u,v\})$:
Accept.



Extra Example: Long Cycle

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Accept.

iii. Reject.

Extra Example: Long Cycle

ii. For all $u,v \in$ **R**:

a) $\boldsymbol{P}$ - shortest (u,v)-path in G[**R**](BFS).

If $|V(\boldsymbol{P})| \neq k$: Next iteration.

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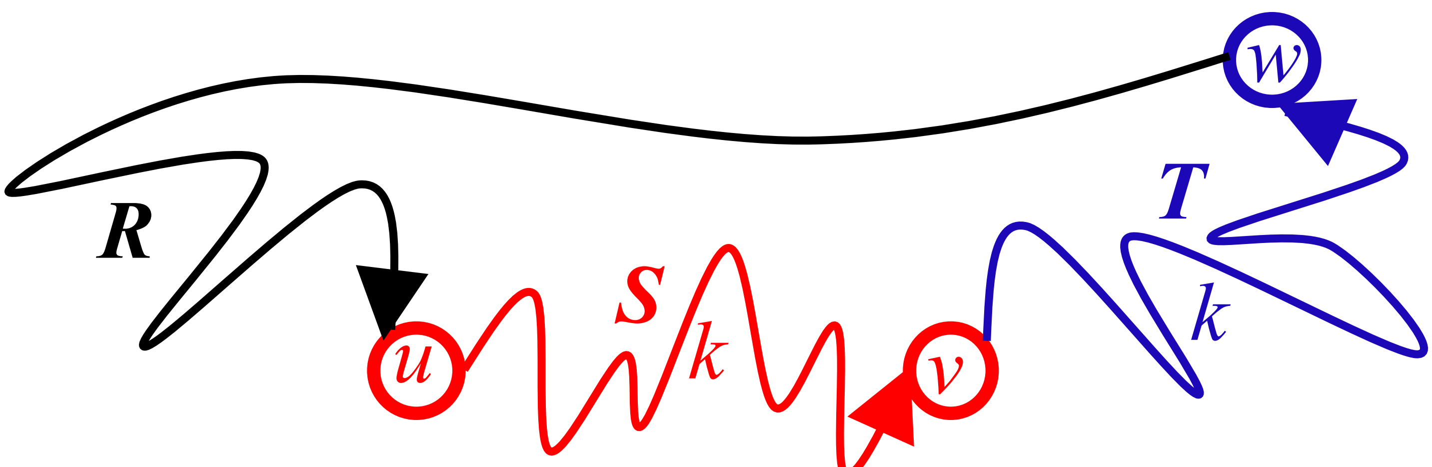
A shortest cycle on at least k vertices:



Extra Example: Long Cycle

ii. For all $u,v \in$ **R**:
   a) $\boldsymbol{P}$ - shortest (u,v)-path in G[**R**](BFS).
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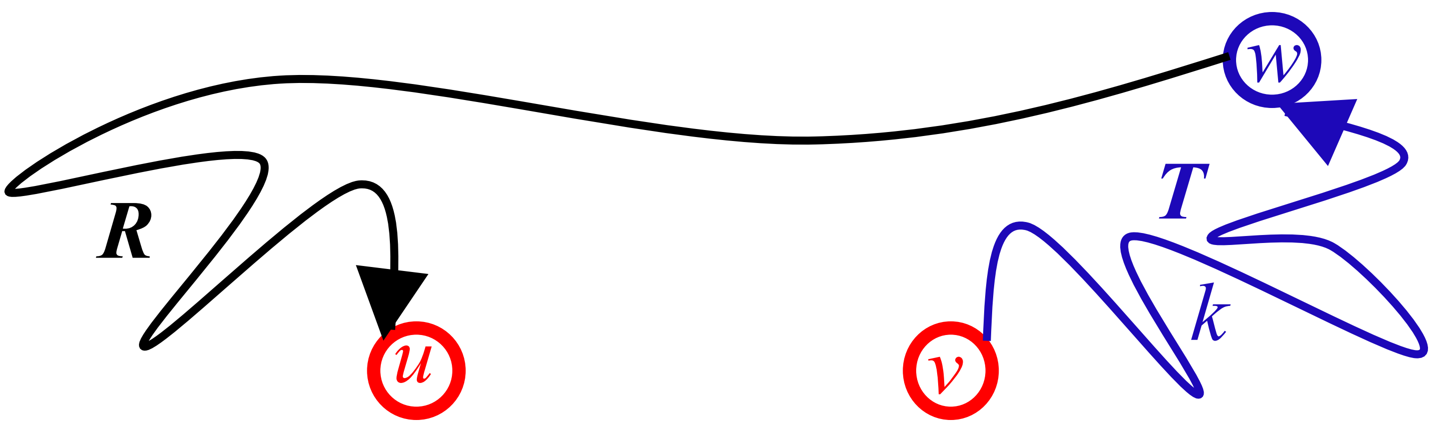
   b) If there is a (v,u)-path in $G \setminus (V(\boldsymbol{P}) \setminus \{u,v\})$: Accept.



Extra Example: Long Cycle

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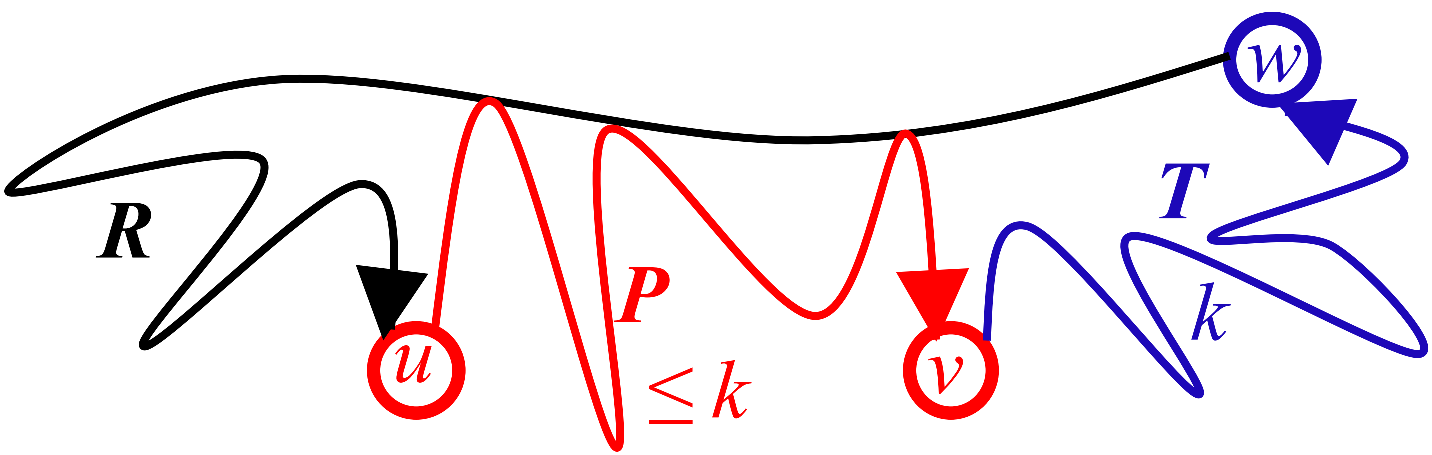

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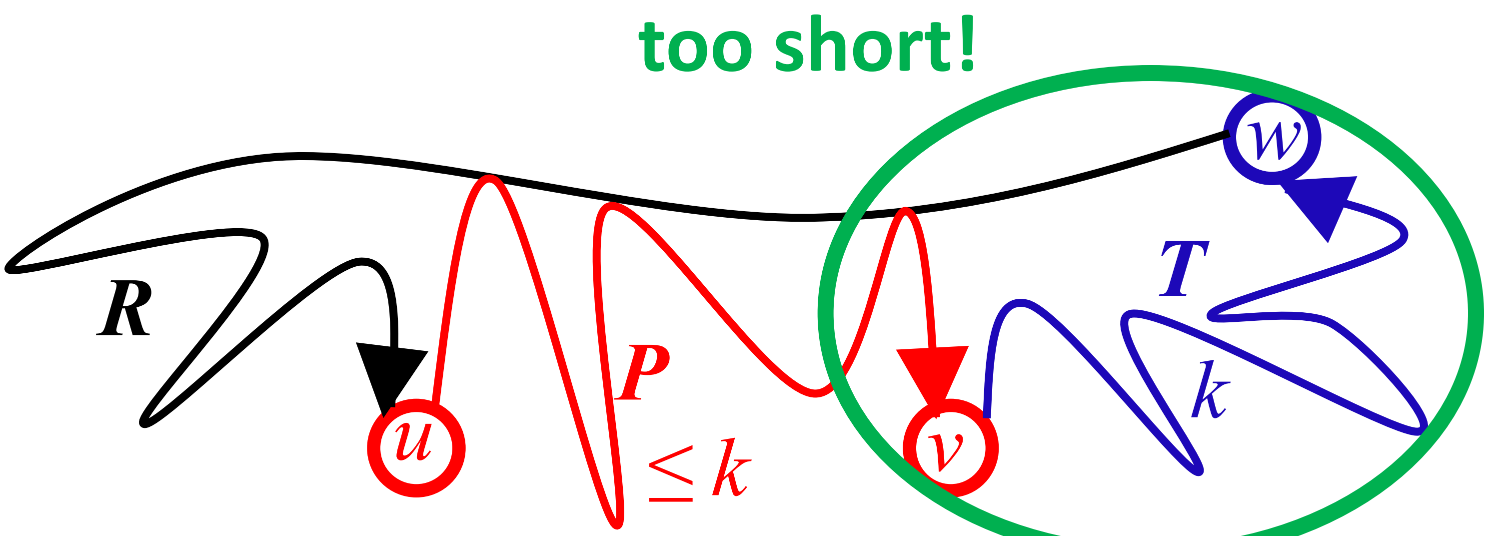

Extra Example: Long Cycle

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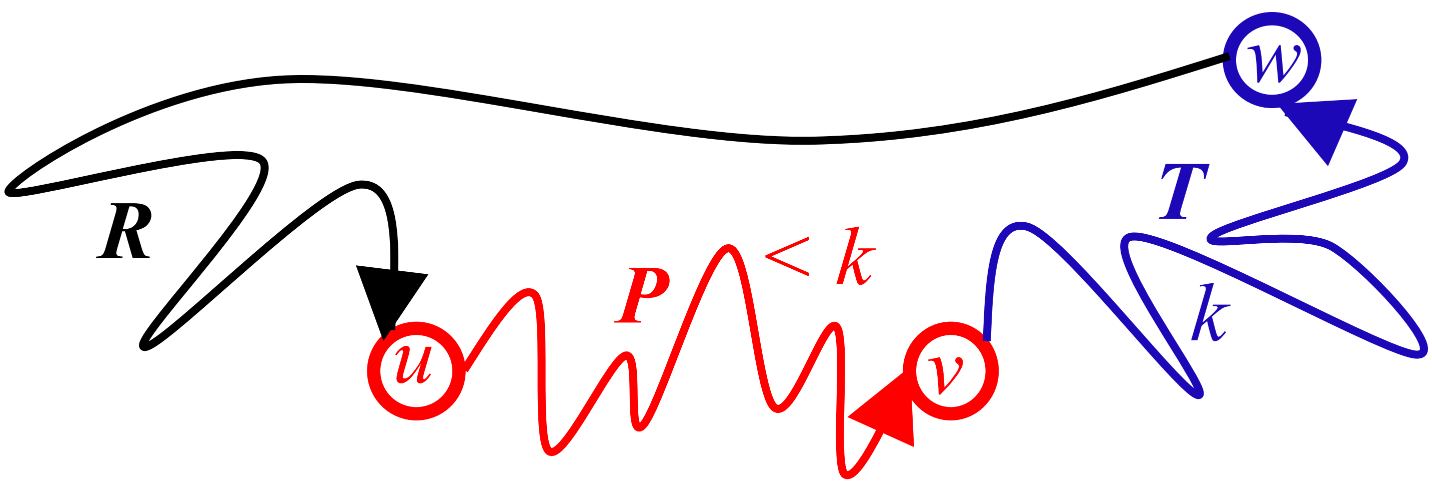

Extra Example: Long Cycle

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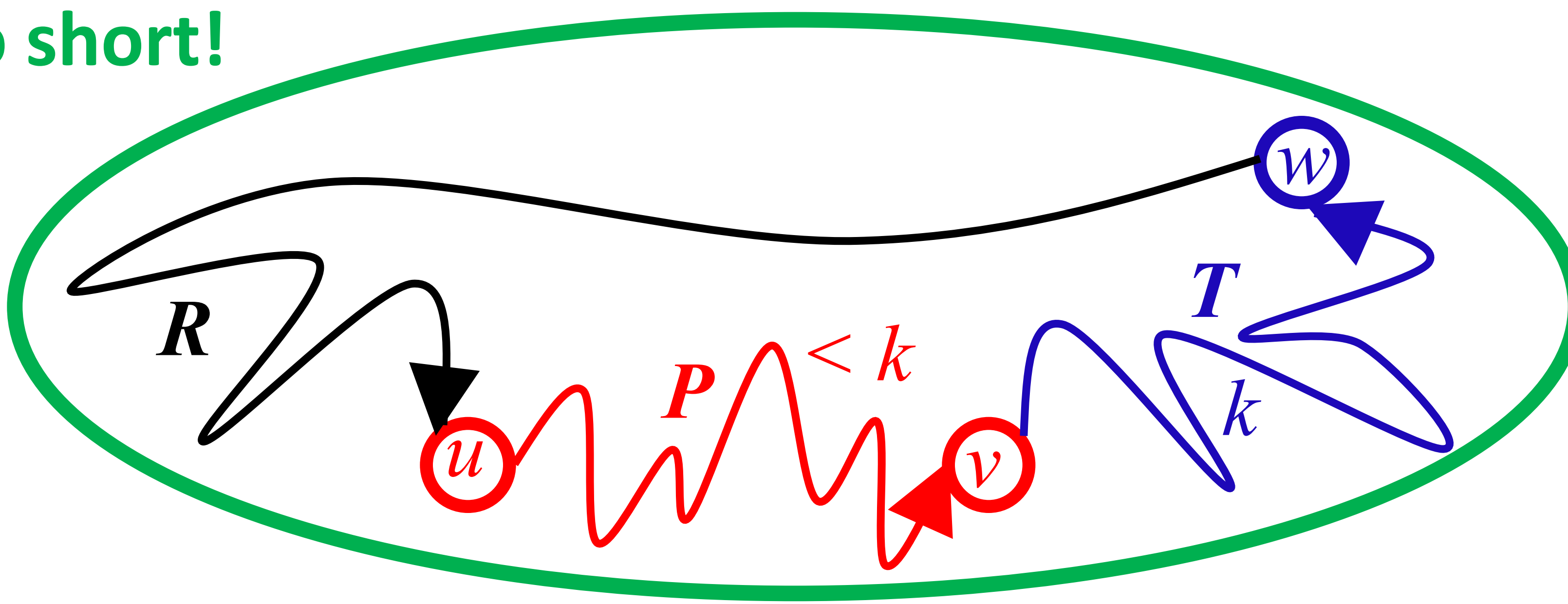
too short!



(Art?) Tutorial

1. **Brute-Force**
2. **Highlights**
3. **Color Coding**
4. **Divide-and-Color**
5. Representative Sets
6. **Mixing**

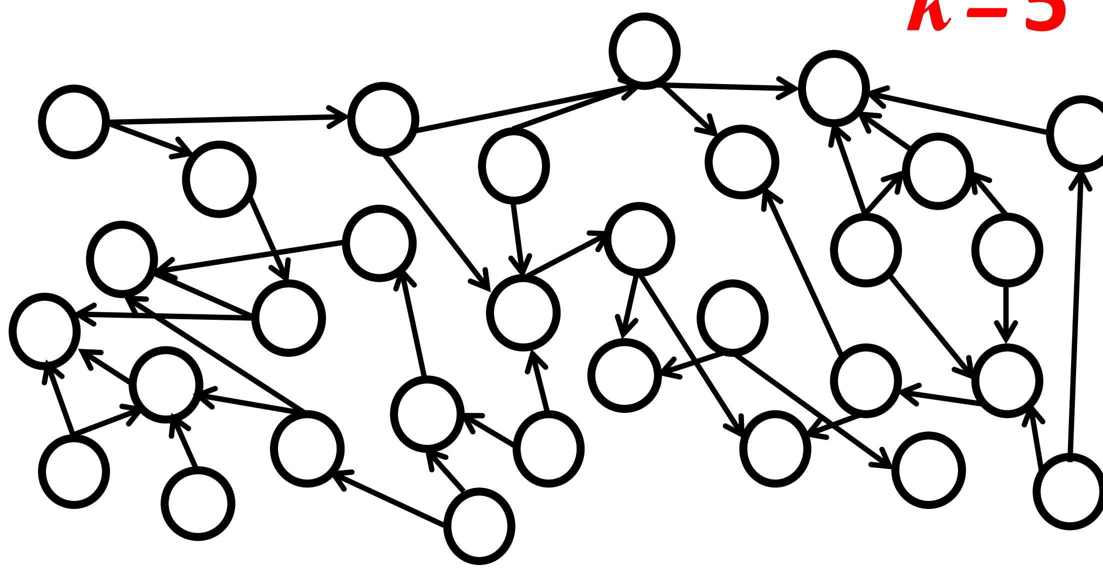
Fomin, Lokshtanov, Panolan and Saurabh, '16

k-Path: Representative Sets

Goal:

DP: add one vertex at a time (color coding).

$k=5$

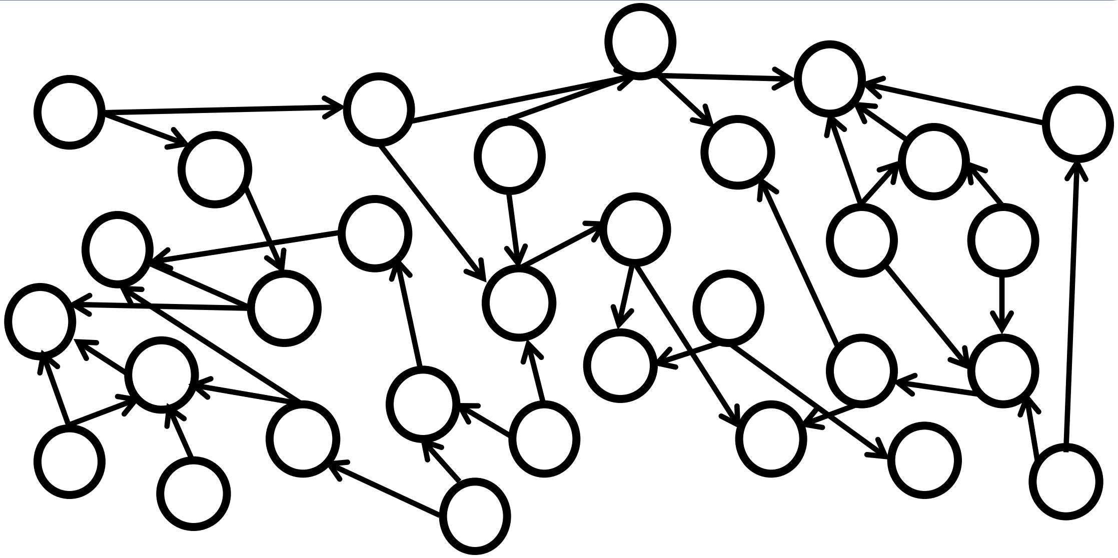


k-Path: Representative Sets

Goal:

DP: add one vertex at a time (color coding).

Tool: erase redundancy; new step → new application (div-and-col).



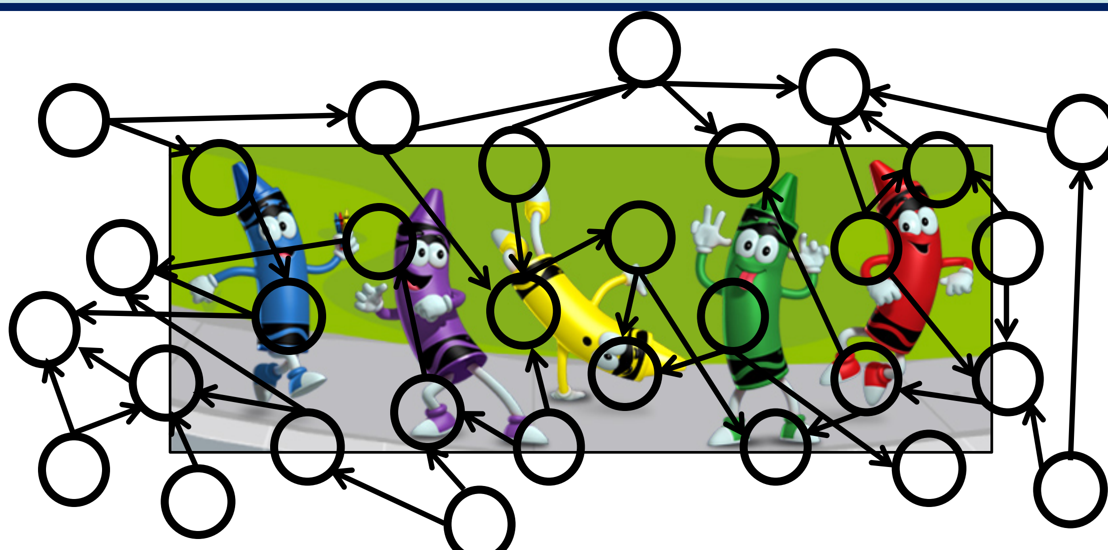
k-Path: Representative Sets

Goal:

DP: add one vertex at a time.

Tool: erase redundancy.

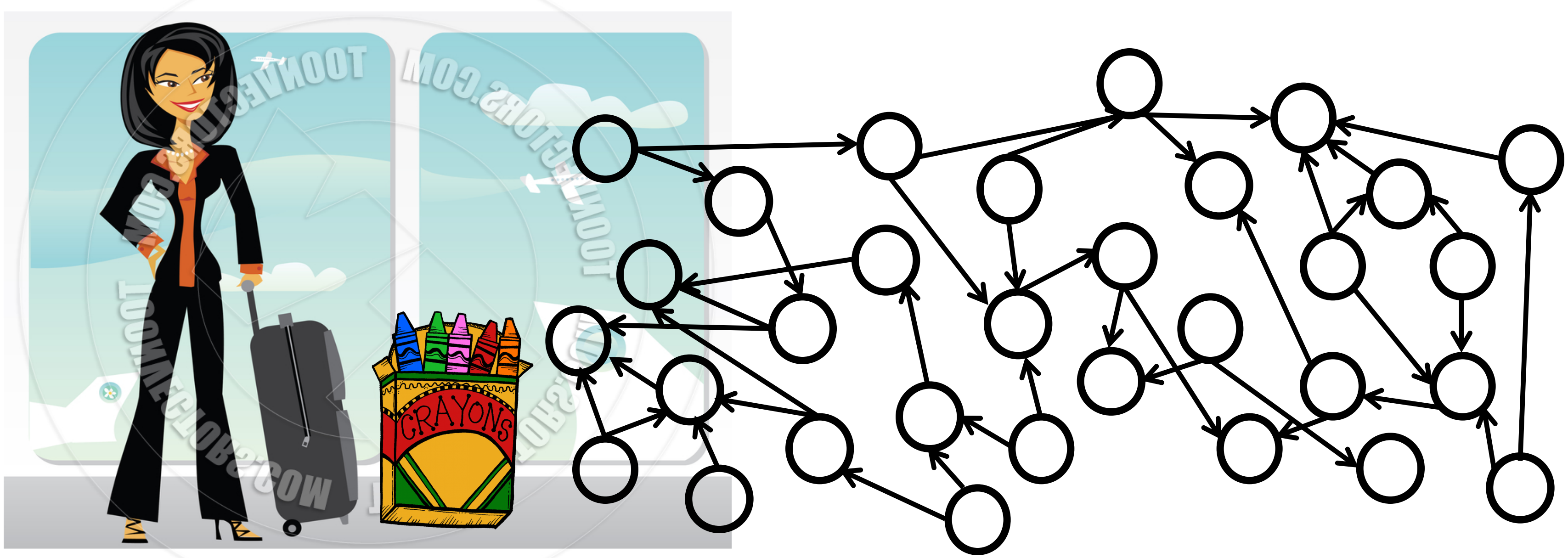
Coloring?



k-Path: Representative Sets

Coloring?

Implicit in the proof of the construction of the tool (black box).

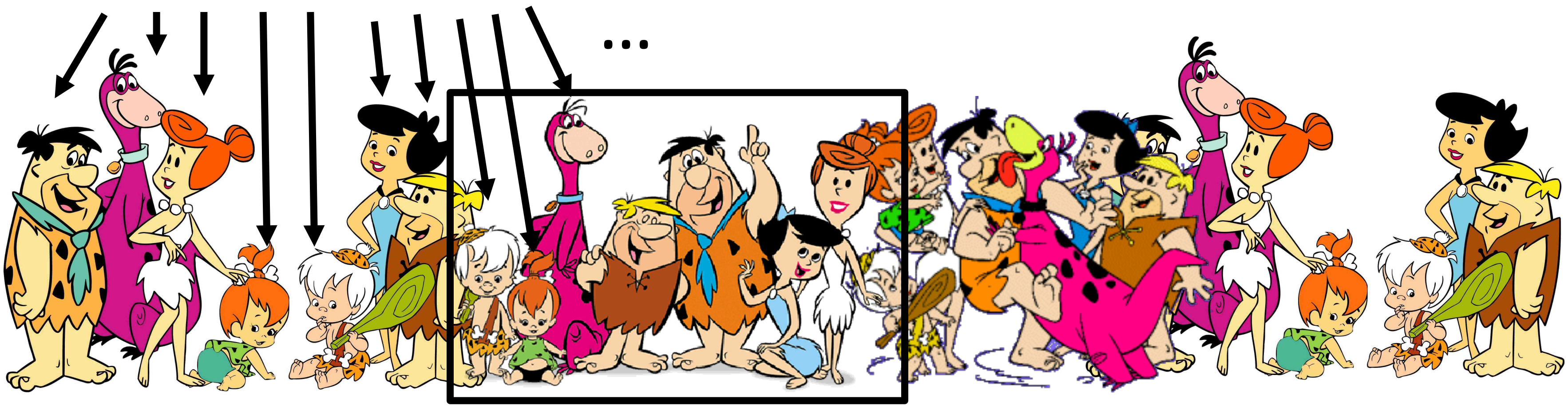


k-Path: Representative Sets

Tool: erase redundancy.

Computation of a **representative family**.

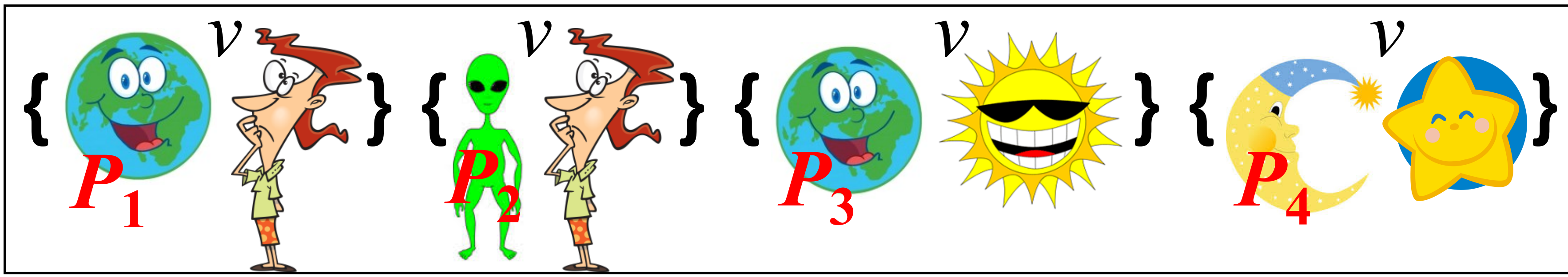
Partial solutions



k-Path: Representative Sets

What is redundant? $k = 5; \; n = 7$

Partial solutions: 3-paths ending at v.



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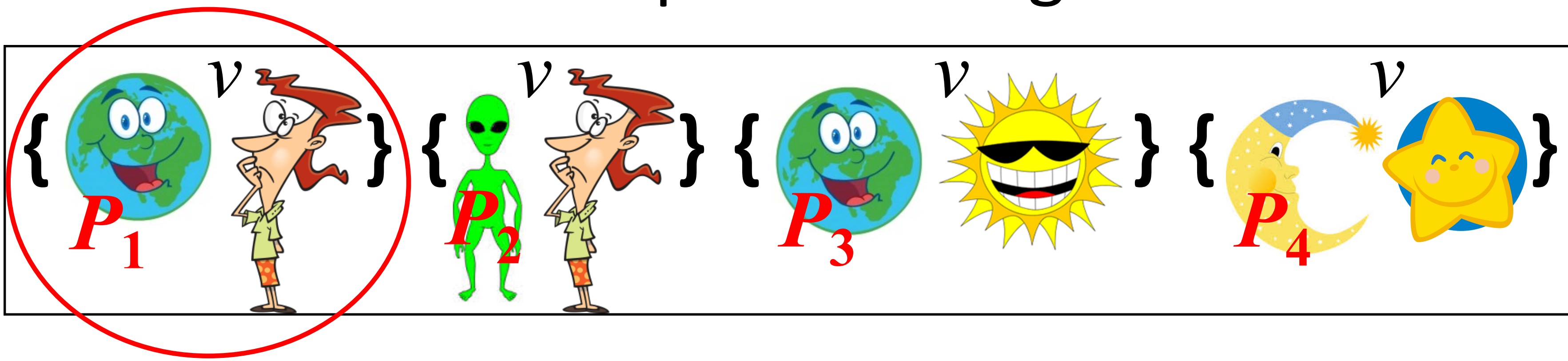

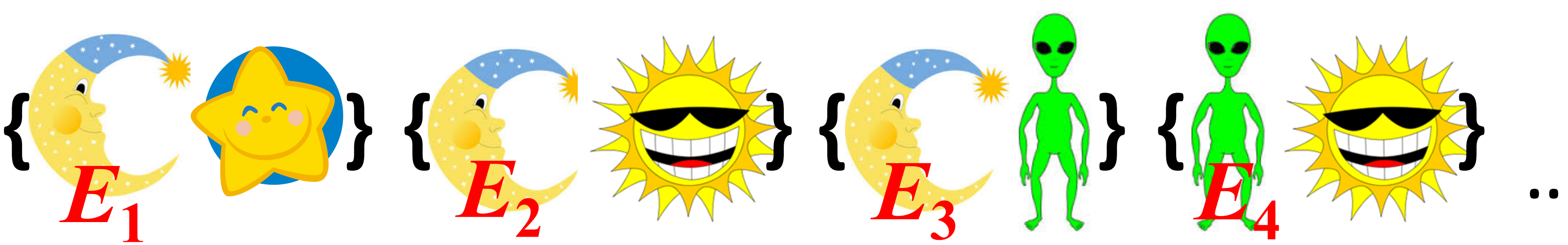

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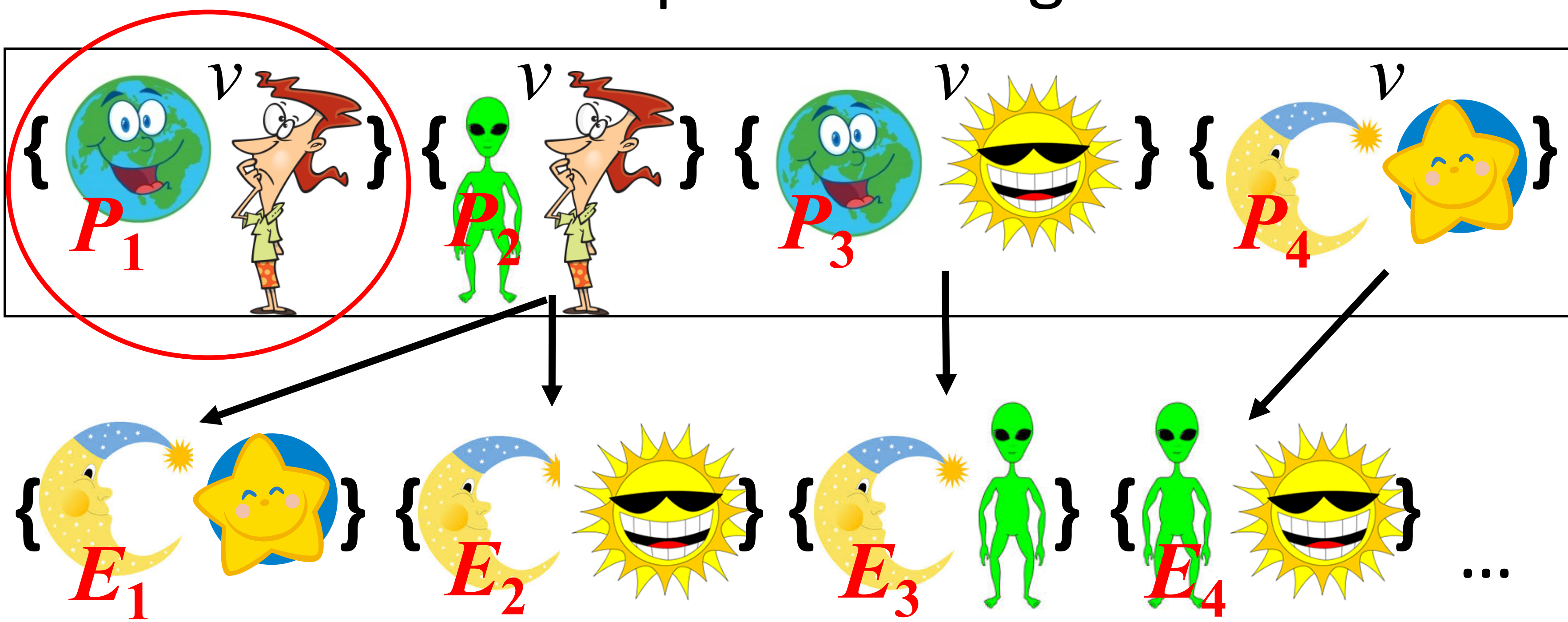

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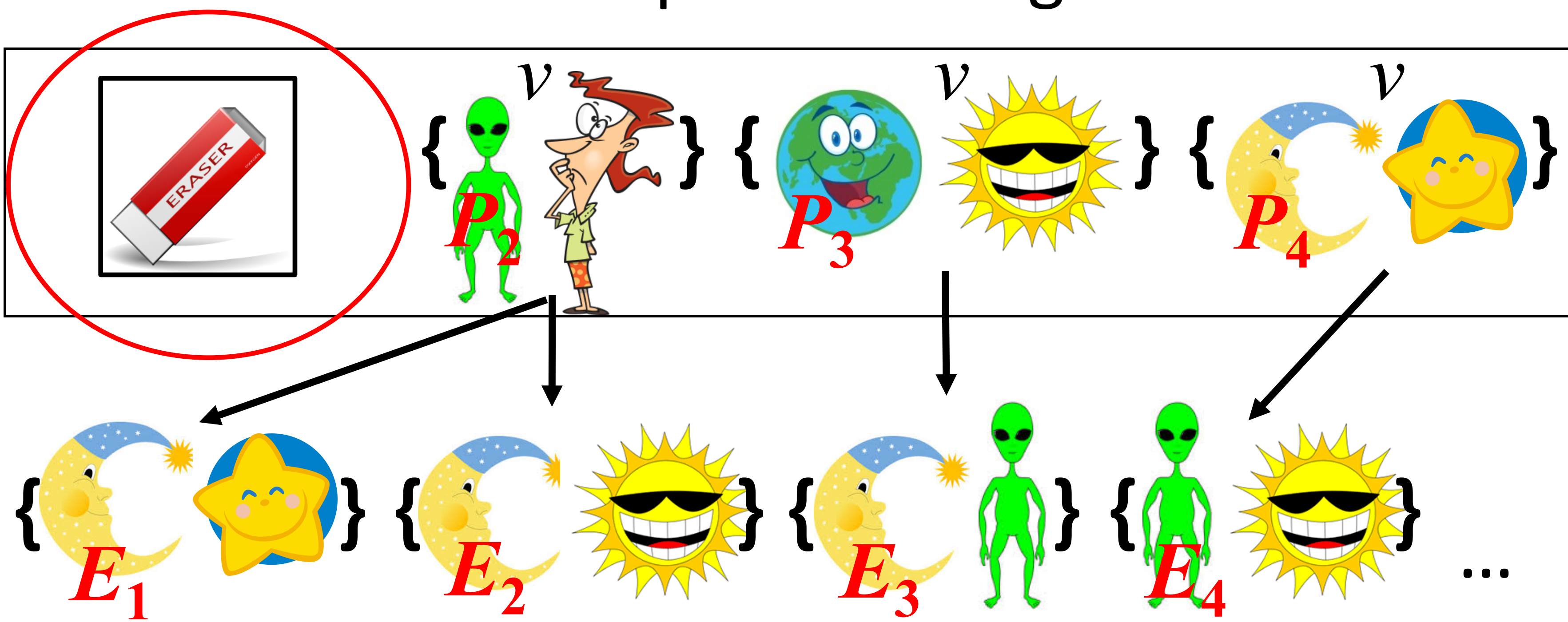


k-Path: Representative Sets

Representative family:

Let S be a family of p-sets.

A subfamily S' of S **k-represents** S if:

For all disjoint $X \in S$ and $Y \subseteq V$ of size $k-p$,

there exists $X' \in S'$ disjoint from Y.

k-Path: Representative Sets

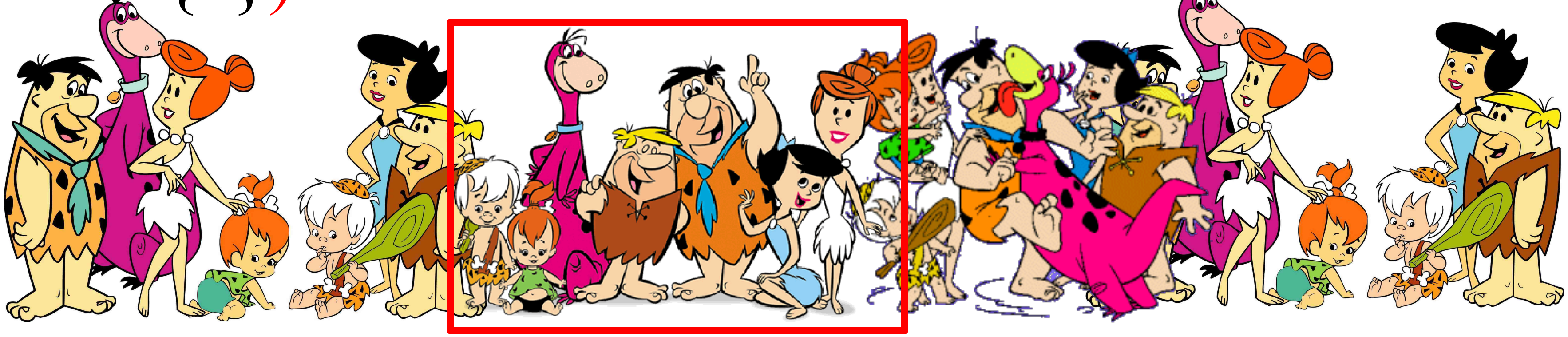
DP:

- $M[v,p]$: The family of vertex-sets of paths on p vertices that end at v.
- $M[v,p] = \bigcup_{(u,v)\in E} M[u,p\text{-}1]+\{v\}$.

k-Path: Representative Sets

DP:

- $M[v,p]$: Representative family of the family of vertex-sets of paths on p vertices that end at v.
- $M[v,p]$ = k-represent($\bigcup_{(u,v) \in E} M[u,p\text{-}1] + \{v\}$).

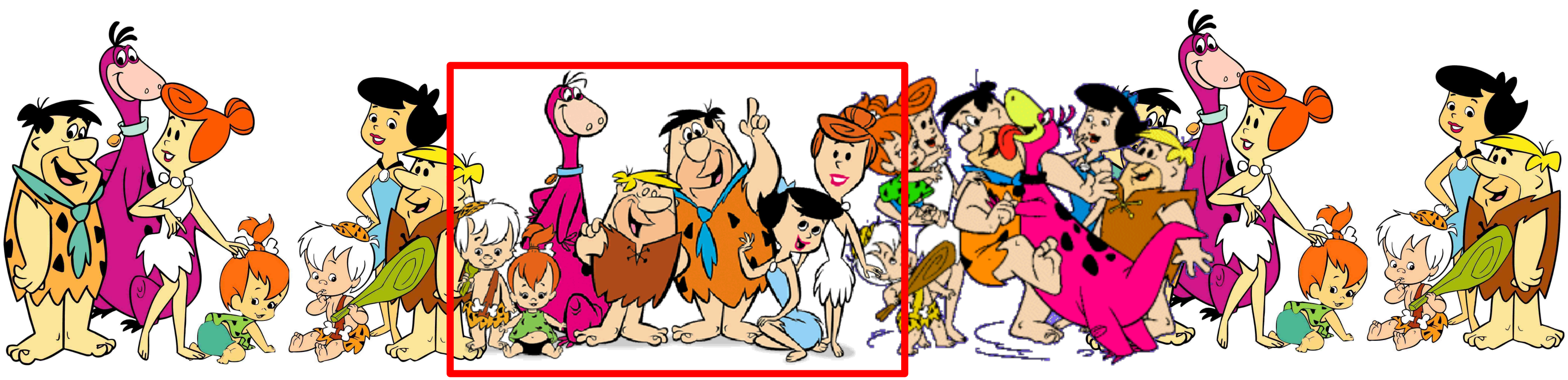


k-Path: Representative Sets

$M[v,p]$ = k-represent($\bigcup_{(u,v)\in E} M[u,p\text{-}1] + \{v\}$).

Running time: [randomized/deterministic]

k-representative family of size $\binom{k}{p}2^{o(k)}\log n$ can be computed in time $O(|\mathcal{S}|(k/(k-p))^{k-p}2^{o(k)}\log n)$.



k-Path: Representative Sets

$$\mathrm{M}[v,p] = k\text{-represent}\Big(\bigcup_{(u,v)\in E} \mathrm{M}[u,p\text{-}1] + \{v\}\Big).$$

Running time: [randomized/deterministic]

k-representative family of size $\binom{k}{p}\underline{2^{o(k)}}\log n$ can be computed in time $O(|\mathcal{S}|(k/(k-p))^{k-p}\underline{2^{o(k)}}\log n)$.

$$O^*(\max_p\{|\mathrm{M}[\cdot,p\text{-}1]|\cdot(k/(k\text{-}p))^{k\text{-}p}\underline{2^{o(k)}}\})$$

k-Path: Representative Sets

$$M[v,p] = k\text{-represent}\Big(\bigcup_{(u,v)\in E} M[u,p\text{-}1] + \{v\}\Big).$$

Running time: [randomized/deterministic]

k-representative family of size $\binom{k}{p} 2^{o(k)} \log n$ can be computed in time $O(|\mathcal{S}|(k/(k-p))^{k-p} 2^{o(k)} \log n)$.

$$O^*(\max_p \{ \binom{k}{p-1} 2^{o(k)} \cdot (k/(k\text{-}p))^{k\text{-}p} 2^{o(k)} \})$$

$$= O^*(2.851^k)$$

k-Path: Representative Sets

$$M[v,p] = k\text{-represent}\left(\bigcup_{(u,v)\in E} M[u,p-1] + \{v\}\right).$$

Ru

k- n

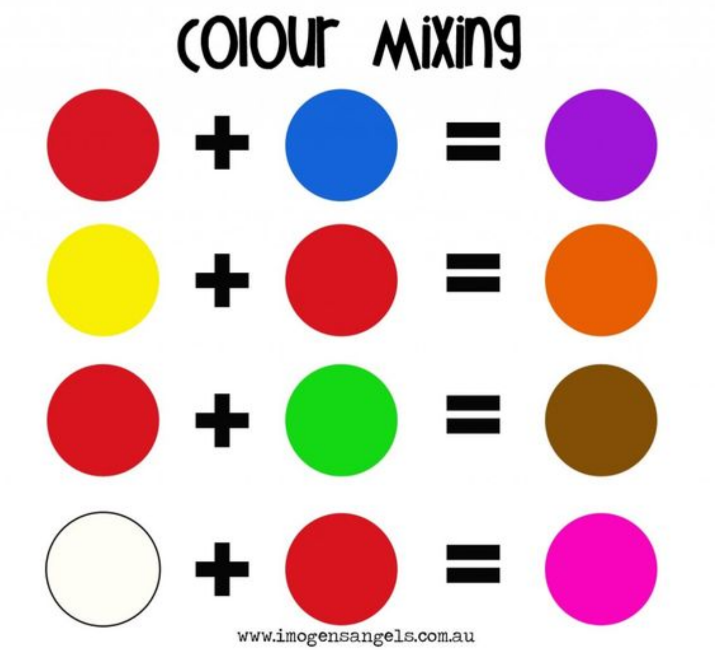
be).

What is the bottleneck of this technique?

$$O^*\left(\max_p\left\{\binom{k}{p-1}\underline{2^{o(k)}}\cdot(k/(k-p))^{k-p}\underline{2^{o(k)}}\right\}\right)$$

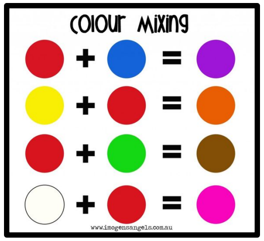
$$= O^*(2.851^k)$$

(Art?) Tutorial



1. Brute-Force
2. Highlights
3. Color Coding
4. Divide-and-Color
5. Representative Sets
6. Mixing

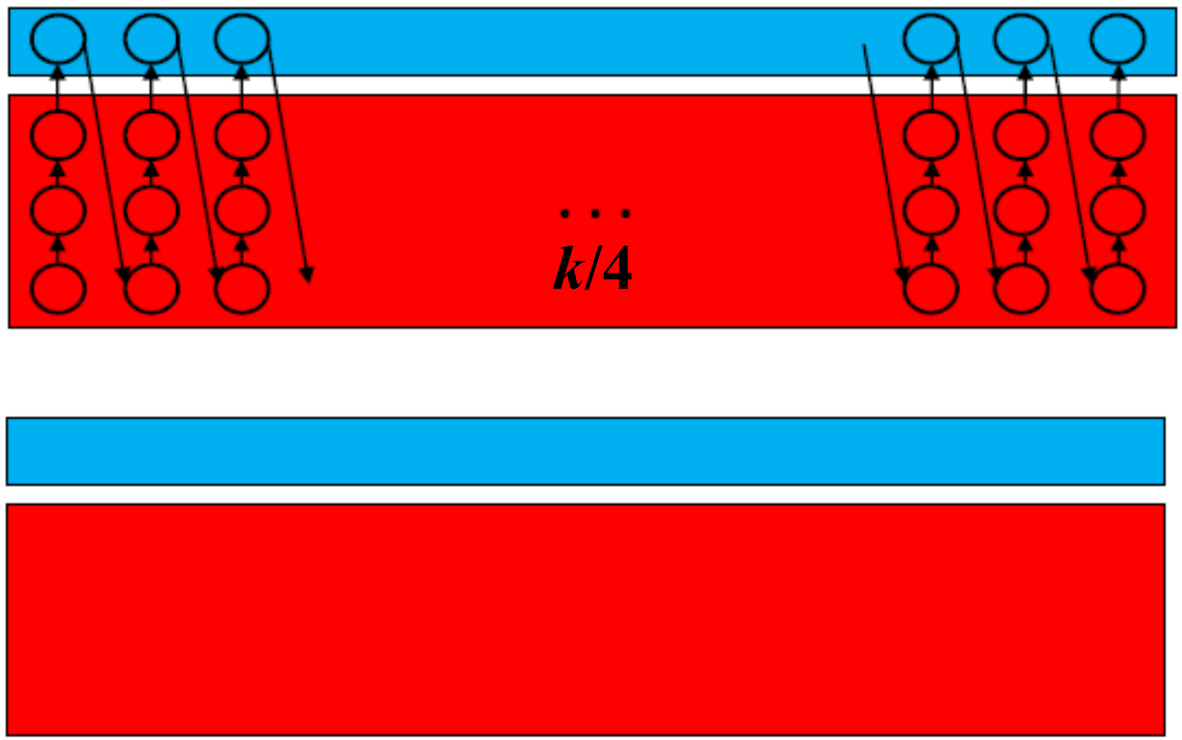
Zehavi, '15



k-Path: Mixing

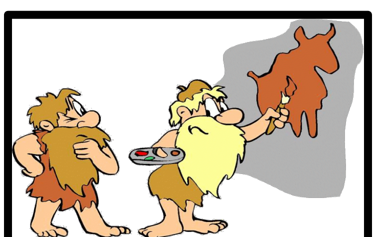
Running time: $O^*(2.597^k)$.

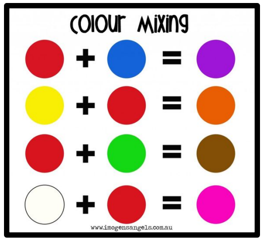
Intuition:



Layer 1.

Correct coloring of a solution.



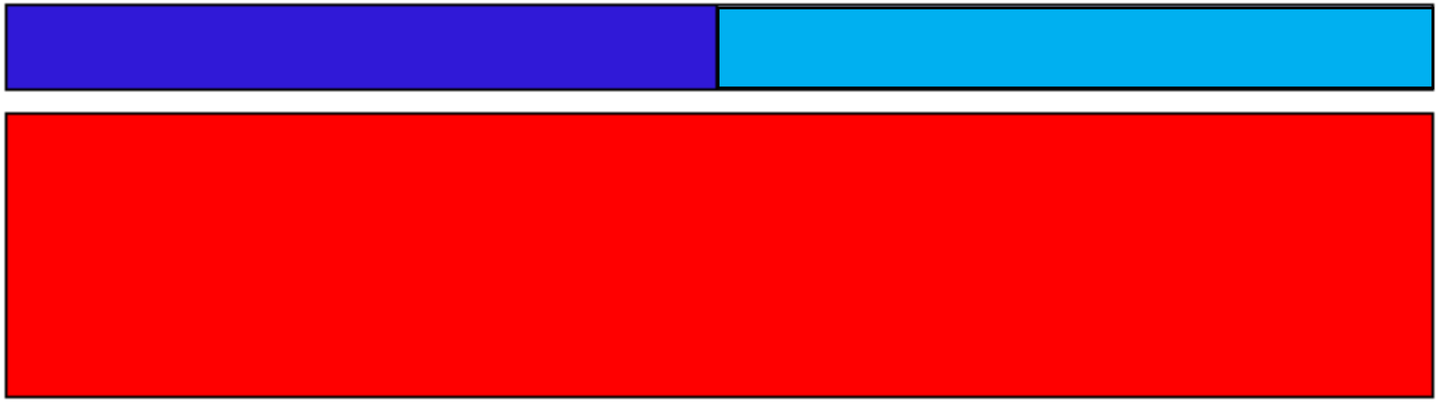


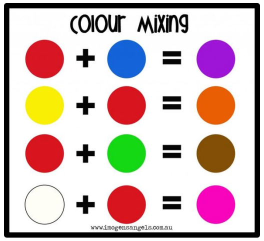
k-Path: Mixing

Running time: $O^*(2.597^k)$.

Intuition:

Layer 2. Correct coloring of a solution.



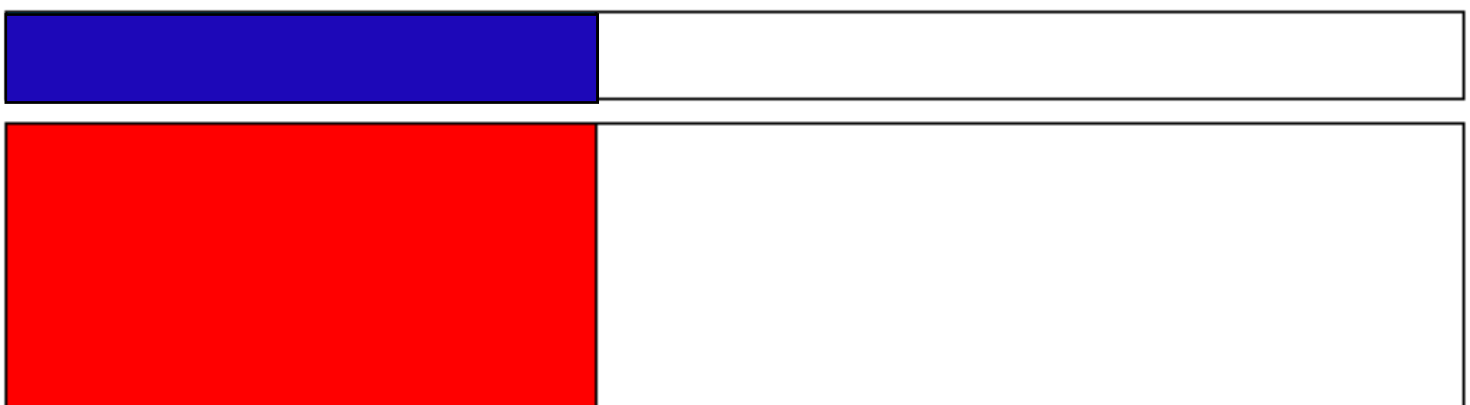


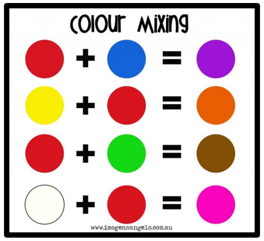
k-Path: Mixing

Running time: $O^*(2.597^k)$.

Intuition:

Layer 3 (DP). Family of p-paths that end at v.



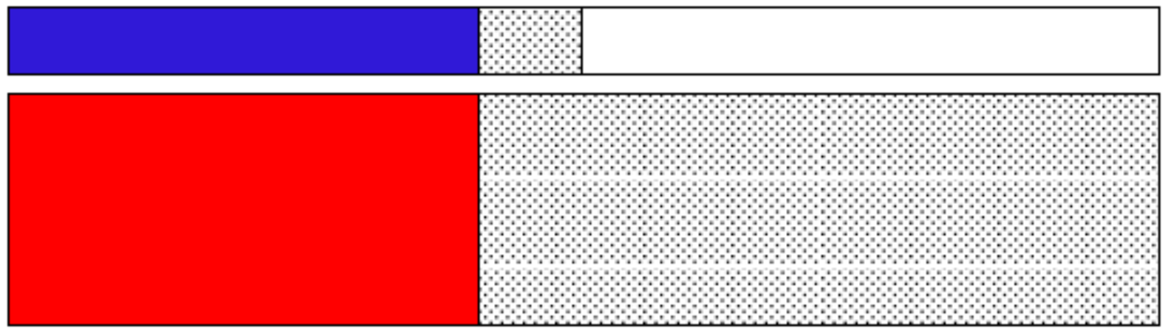


k-Path: Mixing

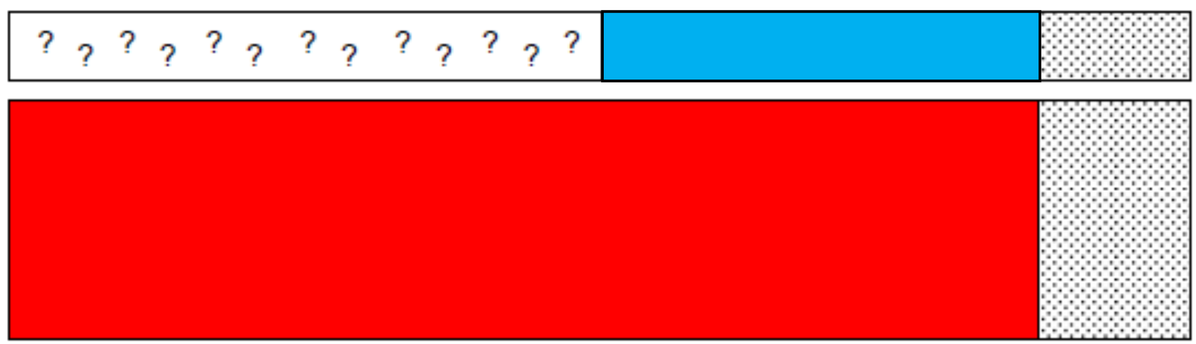
Intuition:

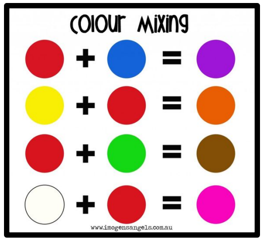
Layer 3 (DP). Family of p-paths that end at v.

First part of the computation:



Second part of the computation:



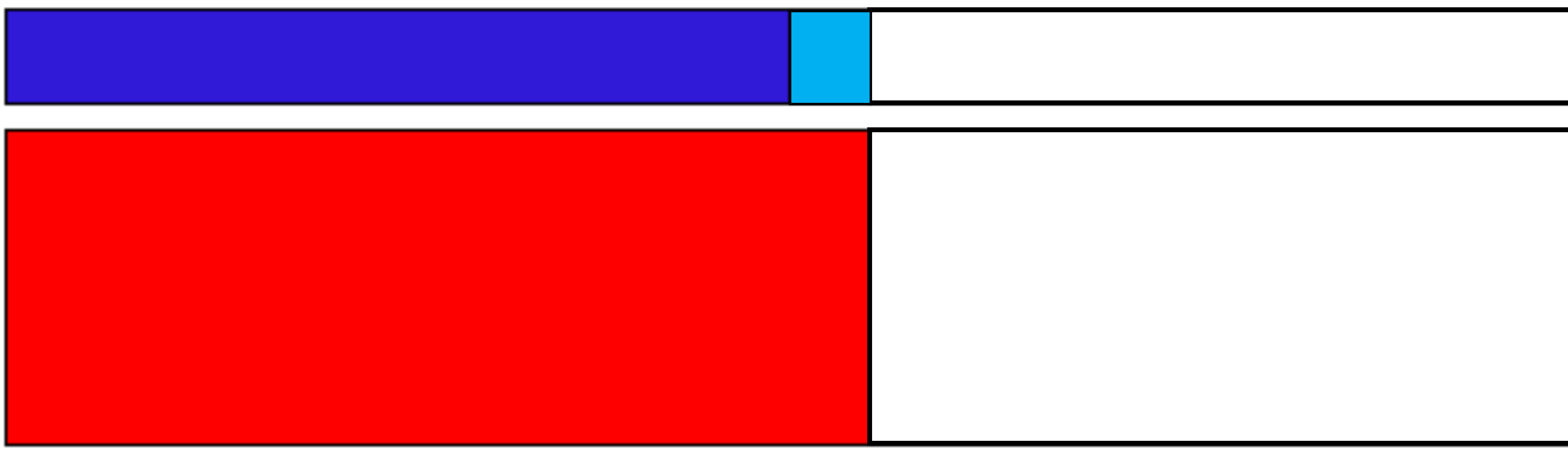


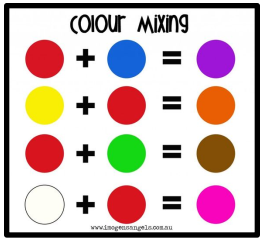
k-Path: Mixing

Intuition:

Layer 3 (DP). Family of p-paths that end at v.

The worst time to compute a representative family:



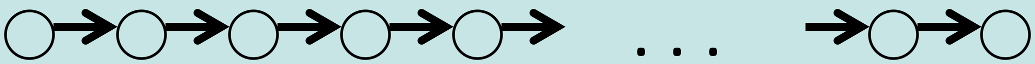


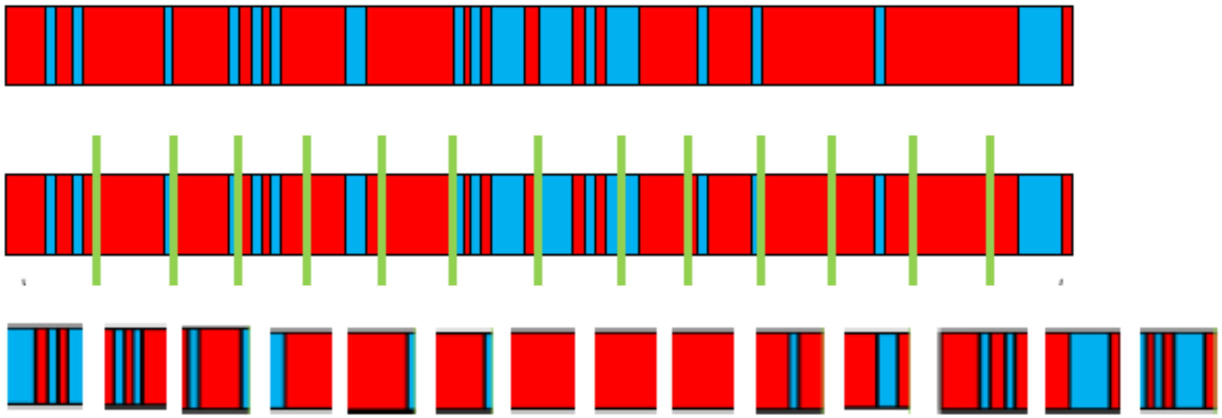
k-Path: Mixing

More general def. + computation of representative sets.

Given the blue set, it is easy to find the dark and light blue sets.

Balanced cutting:





k-Path: Conclusion

- Directed k-Path: highlighting; color coding; divide-and-color; representative sets; mixing.
- Directed Long Cycle.

k-Path: Conclusion

- Directed *k*-Path: highlighting; color coding; divide-and-color; representative sets; mixing.
- Directed Long Cycle.
- Other problems: 3-Set *k*-Packing, 3D *k*-Matching, subcases of Subgraph Isomorphism, Graph Motif, Partial Cover , *k*-Internal Out-Branching, ...

k-Path: Conclusion

- Directed k-Path: highlighting; color coding; divide-and-color; representative sets; mixing.
- Directed Long Cycle.

Open problems:

- Directed k-Path: $O^*(2^k)$ (deterministic).
- Directed Long Cycle: $O^*(4^k)$ (deterministic).
- Directed k-Path: $O^*((4-\varepsilon)^k)$ (deterministic; polynomial space).

Thank you for your attention.

Questions?