Pointer-based Data Structures (2/5)

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Dynamic Sets

	BST	RB-tree	Scapegoat tree	Splay tree
SEARCH(S,k)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
MINIMUM(S)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
MAXIMUM(S)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
SUCCESSOR(S,x)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
PREDECESSOR(S,x)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
INSERT(S, x)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
DELETE(S, x)	$O(h)$	$O(\log n)$	$O(\log n)$	$O(\log n)$

Lec. 1 & 2

— worst-case time

— amortized worst-case time

Exercises

- Prove that the amortized cost of a "zig-zag" operation at a node x is $3(r'(x) - r(x))$
- Think of a following modification of the splay tree: instead of splaying a node each time we access it, move it to root by a number of standard rotations ("zigs"). Give a BST T of size n and a sequence of n searches in T that would require $\omega(n \log n)$ time with such procedure. How bad can it be?
- Does red-black tree have sequential access property? (Sequential access property is: if we search for the n keys stored in the tree in increasing order, i.e., first search for the smallest, second for the second smallest and so on, then we spend time $O(n)$). Justify your answer.
- Does scapegoat tree have sequential access property?

Reading on Splay trees

- (amortized analysis) Cormen et al, Chapter 17
- Jeff Erikson's lecture notes: http://jeffe.cs.illinois.edu/teaching/datastructures/notes/02-scapegoat-splay.pdf
- (properties of splay tree, dynamic optimality) Erik Demaine's lecture notes and video: https://courses.csail.mit.edu/6.851/fall17/scribe/lec5.pdf and video https://courses.csail.mit.edu/6.851/fall17/lectures/L05.html
- Pat Morin's lecture notes: https://cglab.ca/ morin/teaching/5408/notes/splay.pdf
- М. Бабенко, М. Левин "Введение в теорию алгоритмов и структур данных"МЦНМО 2016.
- Robert E. Tarjan. Data Structures and Network Algorithms. CBMS-NSF Reg. Conference Series in Appl. Math. 44. SIAM, 1983.
- original: Daniel D. Sleator and Robert E. Tarjan. Self-adjusting binary search trees. J. ACM 32(3):652–686, 1985.
- original2: Robert E. Tarjan. Sequential access in splay trees takes linear time. Combinatorica 5(5):367–378, 1985.