The OpenSMT Solver

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Outline

Introduction

- Satisfiability Modulo Theory (SMT) Solvers are key engines of several verification approaches

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- Efficient solvers however are **proprietary** (Z3, Yices, Barcelogic, MathSAT, ...)

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- Satisfiability Modulo Theory (SMT) Solvers are key engines of several verification approaches
- Efficient solvers however are proprietary (Z3, Yices, Barcelogic, MathSAT, ...)
- OpenSMT is an effort of providing a simple, extensible, and efficient infrastructure for the development of customized decision procedures

Introduction

- Satisfiability Modulo Theories combines the efficiency of SAT and theory-specific decision procedures

$$a \wedge ((x + y \leq 0) \vee \neg a) \wedge ((x = 1) \vee b)$$

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$$a \wedge (\underbrace{(x + y \leq 0)}_{c} \vee \neg a) \wedge (\underbrace{(x = 1)}_{d} \vee b)$$

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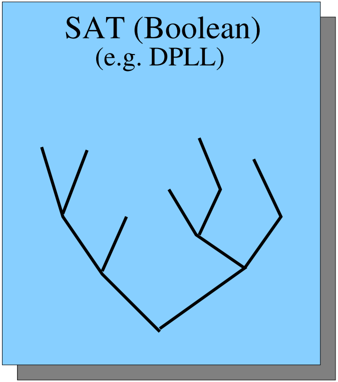

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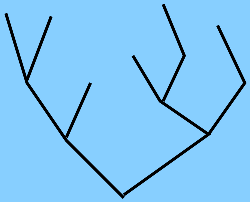
- We need to reason about Boolean combinations of atoms in a theory T (LRA for instance)

SAT (Boolean)
(e.g. DPLL)



Linear Arithmetic
(e.g. Simplex)

$a_1x_1 + \ldots + a_nx_n + b \leq 0$

______________ ≤ 0

______________ ≤ 0

______________ ≤ 0

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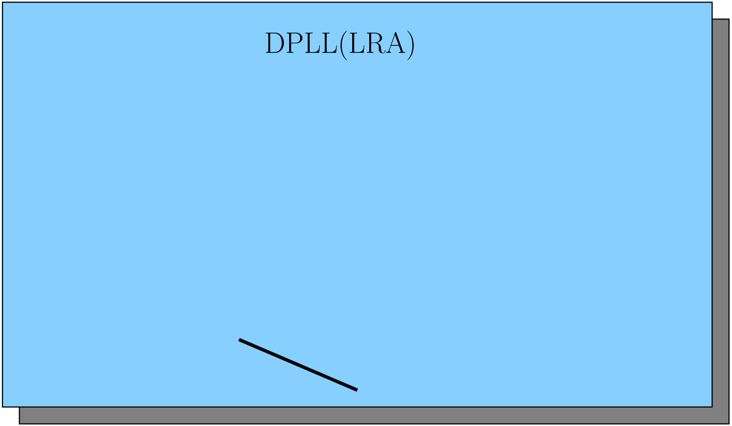

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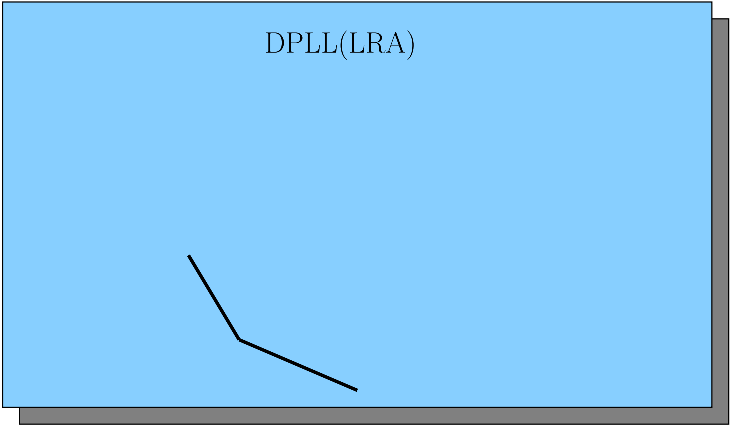

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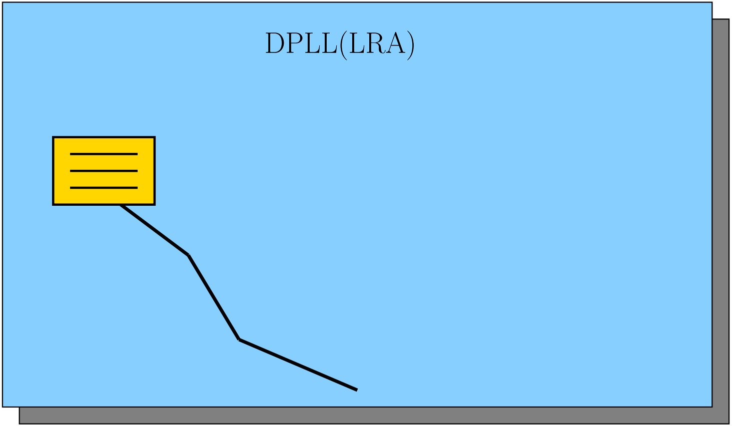

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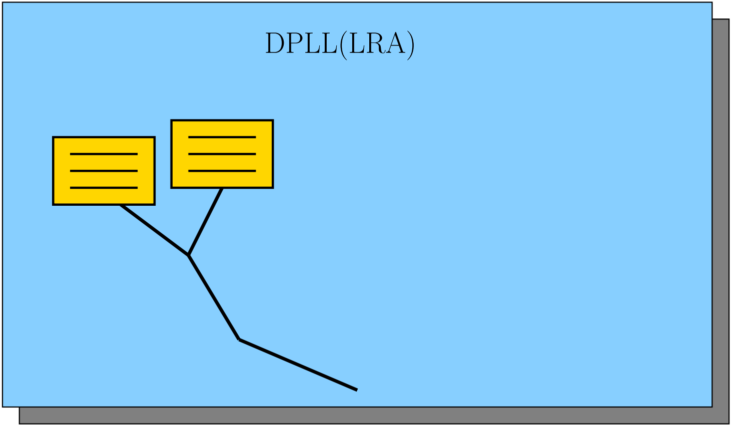

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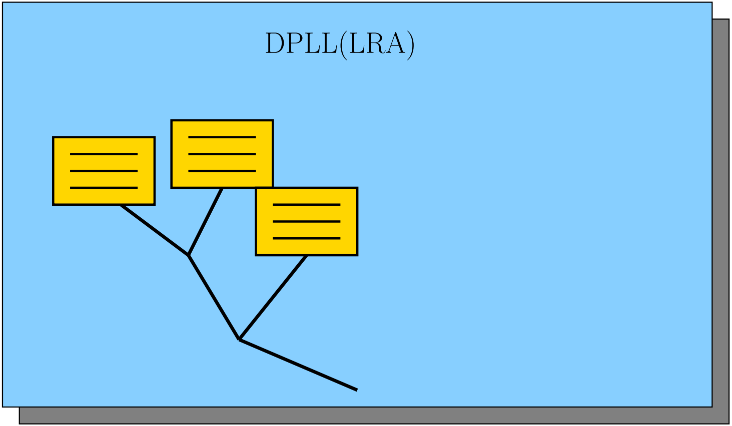

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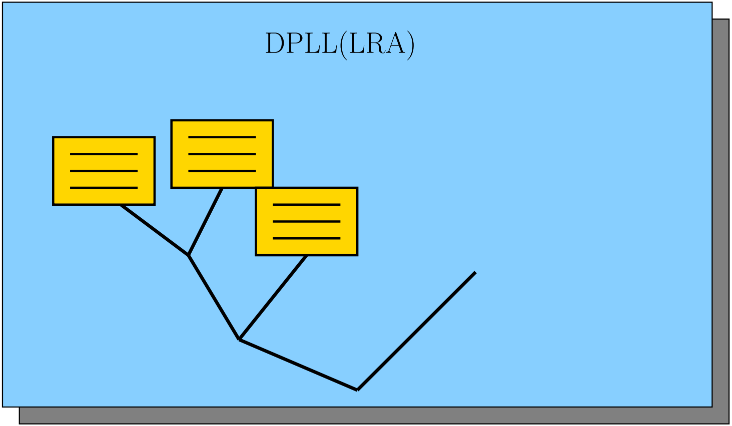

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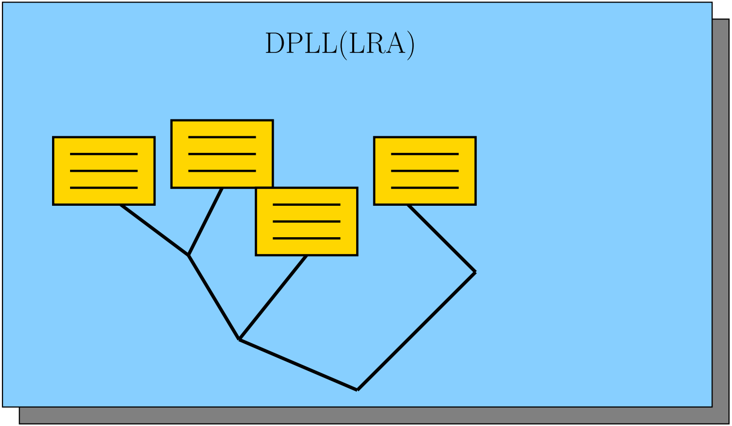

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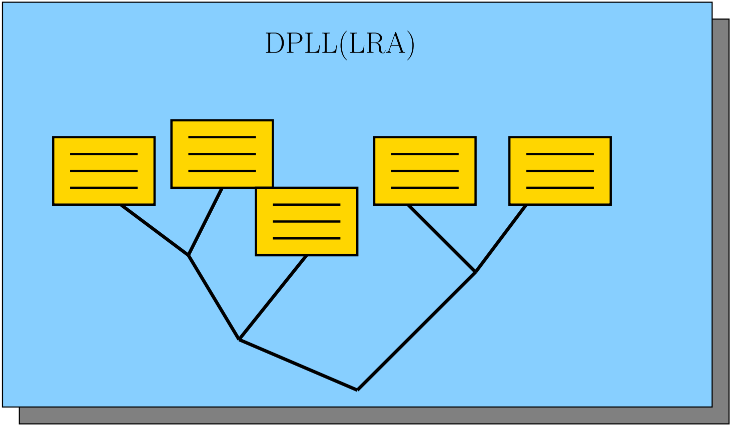

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  - Let DPLL enumerate Boolean models
  - Check LRA constraints with Simplex
- However a lot more has to be done to make it efficient
  - Don't wait for complete Boolean model
  - Theory Propagation
  - Preprocessing
  - Conversion to CNF
  - Theory Layering
  - ...

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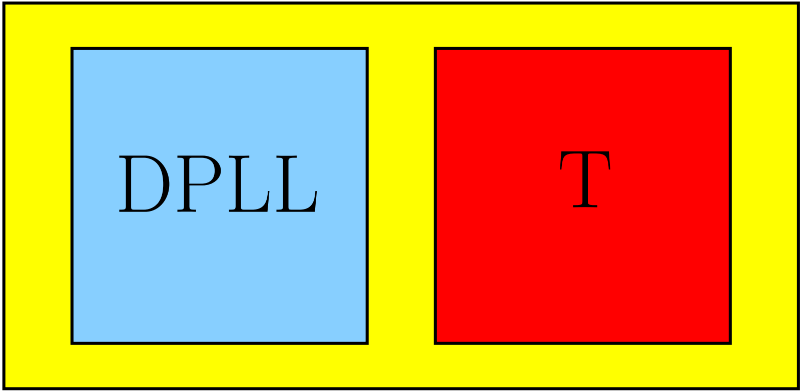
$$e(DPLL(T)) = \boxed{e(DPLL)} + \boxed{e(T)} + \boxed{e(COMM)}$$



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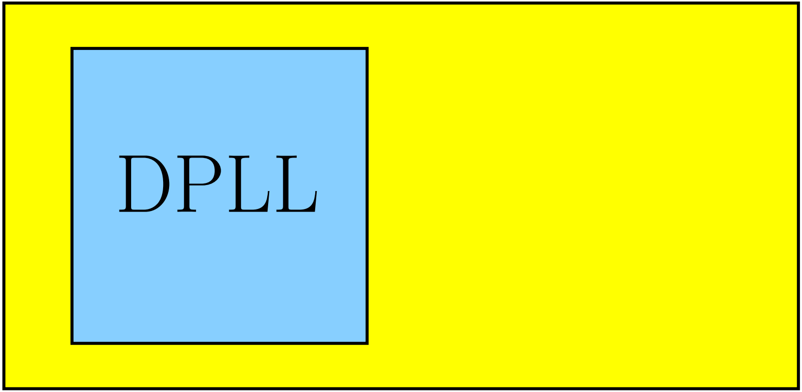

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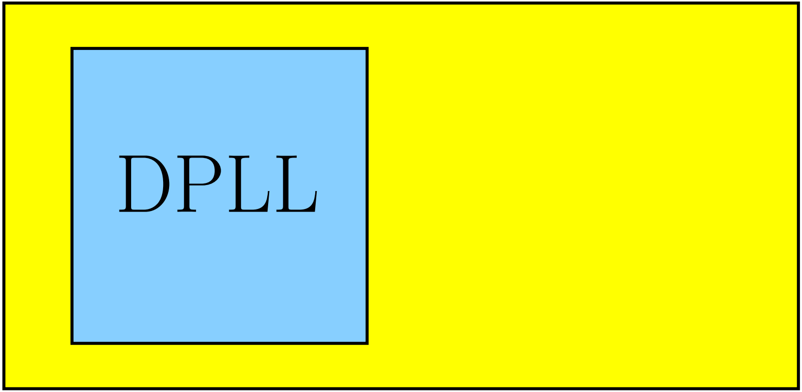
$$e(DPLL(T)) \approx \boxed{e(T)}$$



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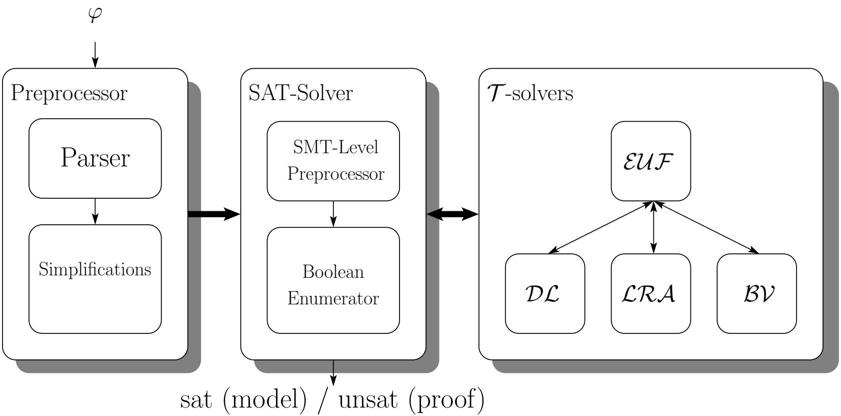
Architecture



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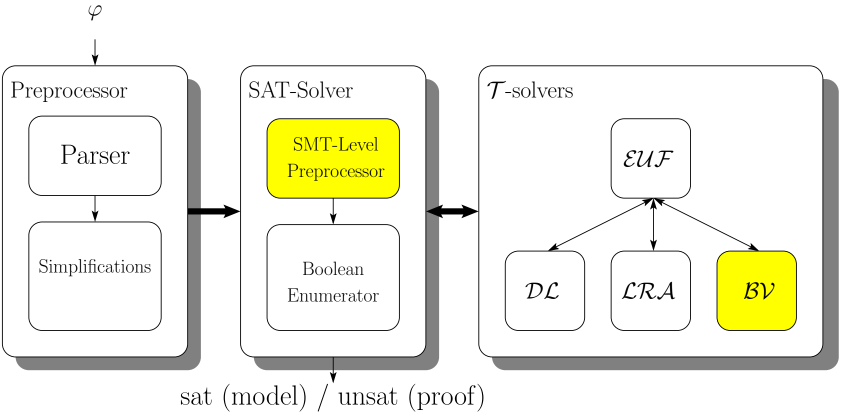

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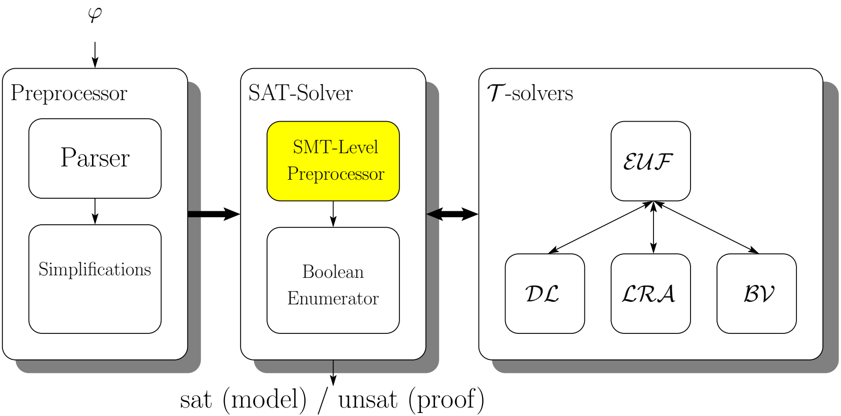

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Outline

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A Generic Template for Variable Elimination Procedures

Variable Types: T_1, T_2, ...
Resolution Rules: R_1, R_2, ...
Algorithm: Input: a set of constraints Repeat Choose a variable X of type T_i to eliminate Combine positive and negative occurrences of X, using R_i

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The Davis-Putnam Procedure [DP60]

Variable Types:
Resolution Rules:
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Variable Types: **Bool**
Resolution Rules: **Boolean Resolution (BR)**
Algorithm: Input: **a set of Boolean clauses** Repeat Choose a variable X of type **Bool** to eliminate Combine positive and negative occurrences of X, using **BR**

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Boolean Resolution

- Clauses are expressions like $(a \vee \neg b \vee c)$, i.e., disjunctions of literals (Boolean variables or negated Boolean variables)

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Boolean Resolution for two clauses

$$(C_1 \vee \mathbf{a} \vee C_2) \otimes_a (D_1 \vee \neg\mathbf{a} \vee D_2) := (C_1 \vee C_2 \vee D_1 \vee D_2)$$

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$$S_a \otimes_a S_{\neg a} := \{\, C_1 \otimes_a C_2 \mid C_1 \in S_a, C_2 \in S_{\neg a} \,\}$$

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$$S_a \otimes_a S_{\neg a} := \{ C_1 \otimes_a C_2 \mid C_1 \in S_a, C_2 \in S_{\neg a} \}$$

Theorem [DP60]

$S_a \cup S_{\neg a}$ is equisatisfiable with $S_a \otimes_a S_{\neg a}$

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DP - Example (on var a)

OLD	NEW
$(a \vee b \vee c)$	
$(a \vee \neg b \vee \neg c)$	
$(\neg a \vee \neg b \vee \neg c)$	
$(\neg a \vee \neg b \vee c)$	

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DP - Example (on var a)

	OLD	NEW
S_a	$(a \vee b \vee c)$ $(a \vee \neg b \vee \neg c)$	
$S_{\neg a}$	$(\neg a \vee \neg b \vee \neg c)$ $(\neg a \vee \neg b \vee c)$	

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S_a	$(a \vee b \vee c)$ $(a \vee \neg b \vee \neg c)$	$(b \vee c \vee \neg b \vee \neg c)$
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$S_{\neg a}$	$(\neg a \vee \neg b \vee \neg c)$	
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S_a	$(a \vee b \vee c)$ $(a \vee \neg b \vee \neg c)$	$(b \vee c \vee \neg b \vee \neg c)$ $(b \vee c \vee \neg b \vee c)$
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$(a \vee b \vee c)$	$(b \vee c \vee \neg b \vee \neg c)$
$(a \vee \neg b \vee \neg c)$	$(b \vee c \vee \neg b \vee c)$
$(\neg a \vee \neg b \vee \neg c)$	$(\neg b \vee \neg c)$
$(\neg a \vee \neg b \vee c)$	$(\neg b \vee \neg c \vee c)$

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DP - Example (on var a)

OLD	NEW
$(a \vee b \vee c)$	$\cancel{(b \vee c \vee \neg b \vee \neg c)}$
$(a \vee \neg b \vee \neg c)$	$\cancel{(b \vee c \vee \neg b \vee c)}$
$(\neg a \vee \neg b \vee \neg c)$	$(\neg b \vee \neg c)$
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The Fourier-Motzkin Elimination [Fou26]

Variable Types:
Resolution Rules:
Algorithm:
Input:
Repeat
Choose a variable X of type to eliminate
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Variable Types: **Rational**
Resolution Rules:
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The Fourier-Motzkin Elimination [Fou26]

Variable Types: **Rational**
Resolution Rules: $\mathcal{LRA}$ **Resolution (RR)**
Algorithm: Input: Repeat Choose a variable X of type to eliminate Combine positive and negative occurrences of X, using

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The Fourier-Motzkin Elimination [Fou26]

Variable Types: **Rational**
Resolution Rules: **$\mathcal{LRA}$ Resolution (RR)**
Algorithm: Input: **a set of $\mathcal{LRA}$ constraints** Repeat Choose a variable X of type to eliminate Combine positive and negative occurrences of X, using

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Algorithm: Input: **a set of $\mathcal{LRA}$ constraints** Repeat Choose a variable X of type **Rational** to eliminate Combine positive and negative occurrences of X, using **RR**

$\mathcal{LRA}$ Resolution

- $\mathcal{LRA}$ constraints are expressions like $3x - 5y + 10z \leq 15$

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$$S_x \otimes_x S_{-x} := \{(x \leq p) \otimes_x (-x \leq q) \mid (x \leq p) \in S_x, (-x \leq q) \in S_{-x}\}$$

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FM - Example (on var z)

	OLD	NEW
	$-x + z \leq -4$	
	$x + z \leq 18$	
	$x - z \leq 6$	
	$-x - z \leq -16$	
	$y \leq 5$	
	$-y \leq -3$	

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FM - Example (on var z)

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S_z	$-x + z \leq -4$ $x + z \leq 18$	
S_{-z}	$x - z \leq 6$ $-x - z \leq -16$	
	$y \leq 5$ $-y \leq -3$	$y \leq 5$ $-y \leq -3$

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	$y \leq 5$	$y \leq 5$
	$-y \leq -3$	$-y \leq -3$

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FM - Example (on var z)

	OLD	NEW
S_z	$-x + z \leq -4$ $x + z \leq 18$	$0 \leq 2$
S_{-z}	$x - z \leq 6$ $-x - z \leq -16$	
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S_{-z}	$x - z \leq 6$	$x \leq 12$
	$-x - z \leq -16$	
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	$x + z \leq 18$	$-x \leq -10$
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FM - Example (on var z)

	OLD	NEW
S_z	$-x + z \leq -4$	$\cancel{0 \leq 2}$
	$x + z \leq 18$	$-x \leq -10$
S_{-z}	$x - z \leq 6$	$x \leq 12$
	$-x - z \leq -16$	$\cancel{0 \leq 2}$
	$y \leq 5$	$y \leq 5$
	$-y \leq -3$	$-y \leq -3$

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FM - Example (on var z)

OLD	NEW
$-x + z \leq -4$	
$x + z \leq 18$	$-x \leq -10$
$x - z \leq 6$	$x \leq 12$
$-x - z \leq -16$	
$y \leq 5$	$y \leq 5$
$-y \leq -3$	$-y \leq -3$

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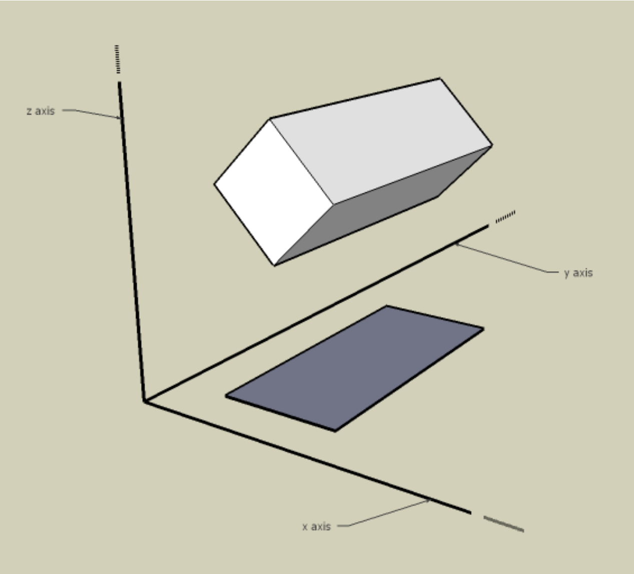
FM - Example (on var z)



DP + FM = DPFM

Variable Types:
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Algorithm: Input: Repeat Choose a variable X of type to eliminate Combine positive and negative occurrences of X, using

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DP + FM = DPFM

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Algorithm: Input: **a set of SMT($\mathcal{LRA}$) clauses** in OCCF Repeat Choose a variable X of type to eliminate Combine positive and negative occurrences of X, using

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Variable Types: **Bool**, **Rational**
Resolution Rules: **BR**, **SMT($\mathcal{LRA}$) Resolution (SR)**

Algorithm:
  Input: **a set of SMT($\mathcal{LRA}$) clauses** in OCCF
  Repeat
    Choose a variable X of type **Bool** (**Rational**) to eliminate
    Combine positive and negative occurrences of X, using

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DP + FM = DPFM

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One Constraint per Clause Form (OCCF)

- DPFM requires clauses in OCCF, clauses that contain **at most one $\mathcal{LRA}$ constraint**
- it is easy to transform clauses in OCCF, by means of auxiliary Boolean variables
  - E.g. $(a \vee (x \leq 3) \vee b \vee (x + y \leq 10))$ can be rewritten as $(a \vee (x \leq 3) \vee b \vee c)$ and $(\neg c \vee (x + y \leq 10))$

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SMT($\mathcal{LRA}$) Resolution

- negated $\mathcal{LRA}$ constr. can be expressed in terms of $\leq$
  - e.g. $\neg(x \leq 10)$ is equiv. to $-x \leq -10 - \delta, \quad (\delta > 0)$ (see [DdM06])

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SMT($\mathcal{LRA}$) Resolution for two clauses in OCCF

$$(C_1 \vee (x \leq p) \vee C_2) \otimes_x (D_1 \vee (-x \leq q) \vee D_2) := C_1 \vee C_2 \vee (-q \leq p) \vee D_1 \vee D_2$$

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$$S_x \otimes_x S_{-x} := \{ C_1 \otimes_x C_2 \mid C_1 \in S_x, C_2 \in S_{-x} \}$$

Theorem

$S_x \cup S_{-x}$ is equisatisfiable with $S_x \otimes_x S_{-x}$

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DPFM - Example (on var z)

$$\begin{array}{ll}
\neg a_1 \vee (-z \leq -3) & a_1 \vee (z \leq 3 - \delta) \vee a_2 \\
\neg a_1 \vee (-x \leq -3) & \neg a_2 \vee (x \leq 3 - \delta) \vee a_3 \\
\neg a_1 \vee (-y \leq -3) & \neg a_3 \vee (y \leq 3 - \delta) \vee a_4 \\
\neg a_1 \vee (y \leq 5) & \neg a_4 \vee (-y \leq 5 - \delta) \vee a_5 \\
\neg a_1 \vee (x \leq 5) & \neg a_5 \vee (-x \leq 5 - \delta) \vee a_6 \\
\neg a_1 \vee (z \leq 5) & \neg a_6 \vee (-z \leq 5 - \delta) \\
\neg b_1 \vee (-z \leq -2) & b_1 \vee (z \leq 2 - \delta) \vee b_2 \\
\neg b_1 \vee (-x \leq -2) & \neg b_2 \vee (x \leq 2 - \delta) \vee b_3 \\
\neg b_1 \vee (-y \leq -2) & \neg b_3 \vee (y \leq 2 - \delta) \vee b_4 \\
\neg b_1 \vee (y \leq 4) & \neg b_4 \vee (-y \leq 4 - \delta) \vee b_5 \\
\neg b_1 \vee (x \leq 4) & \neg b_5 \vee (-x \leq 4 - \delta) \vee b_6 \\
\neg b_1 \vee (z \leq 4) & \neg b_6 \vee (-z \leq 4 - \delta) \\
a_1 \vee b_1 &
\end{array}$$

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DPFM - Example (on var z)

$\neg a_1 \vee (-z \leq -3)$

$\neg a_1 \vee (-x \leq -3)$

$\neg a_1 \vee (-y \leq -3)$

$\neg a_1 \vee (y \leq 5)$

$\neg a_1 \vee (x \leq 5)$

$\neg a_1 \vee (z \leq 5)$

$\neg b_1 \vee (-z \leq -2)$

$\neg b_1 \vee (-x \leq -2)$

$\neg b_1 \vee (-y \leq -2)$

$\neg b_1 \vee (y \leq 4)$

$\neg b_1 \vee (x \leq 4)$

$\neg b_1 \vee (z \leq 4)$

$a_1 \vee b_1$

$a_1 \vee (z \leq 3 - \delta) \vee a_2$

$\neg a_2 \vee (x \leq 3 - \delta) \vee a_3$

$\neg a_3 \vee (y \leq 3 - \delta) \vee a_4$

$\neg a_4 \vee (-y \leq -5 - \delta) \vee a_5$

$\neg a_5 \vee (-x \leq -5 - \delta) \vee a_6$

$\neg a_6 \vee (-z \leq -5 - \delta)$

$b_1 \vee (z \leq 2 - \delta) \vee b_2$

$\neg b_2 \vee (x \leq 2 - \delta) \vee b_3$

$\neg b_3 \vee (y \leq 2 - \delta) \vee b_4$

$\neg b_4 \vee (-y \leq -4 - \delta) \vee b_5$

$\neg b_5 \vee (-x \leq -4 - \delta) \vee b_6$

$\neg b_6 \vee (-z \leq -4 - \delta)$

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DPFM - Example (on var z)

	OLD	NEW
	$\neg a_1 \vee (z \leq 5)$ $\neg b_1 \vee (z \leq 4)$ $a_1 \vee (z \leq 3 - \delta) \vee a_2$ $b_1 \vee (z \leq 2 - \delta) \vee b_2$ $\neg a_6 \vee (-z \leq -5 - \delta)$ $\neg b_6 \vee (-z \leq -4 - \delta)$ $\neg a_1 \vee (-z \leq -3)$ $\neg b_1 \vee (-z \leq -2)$	

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DPFM - Example (on var z)

	OLD	NEW
S_z	$\neg a_1 \vee (z \leq 5)$ $\neg b_1 \vee (z \leq 4)$ $a_1 \vee (z \leq 3 - \delta) \vee a_2$ $b_1 \vee (z \leq 2 - \delta) \vee b_2$	
S_{-z}	$\neg a_6 \vee (-z \leq -5 - \delta)$ $\neg b_6 \vee (-z \leq -4 - \delta)$ $\neg a_1 \vee (-z \leq -3)$ $\neg b_1 \vee (-z \leq -2)$	

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DPFM - Example (on var z)

	OLD	NEW
S_z	$\neg a_1 \vee (z \leq 5)$	
	$\neg b_1 \vee (z \leq 4)$	
	$a_1 \vee (z \leq 3 - \delta) \vee a_2$	
	$b_1 \vee (z \leq 2 - \delta) \vee b_2$	
S_{-z}	$\neg a_6 \vee (-z \leq -5 - \delta)$	
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DPFM - Example (on var z)

	OLD	NEW
S_z	$\neg a_1 \vee (z \leq 5)$ $\neg b_1 \vee (z \leq 4)$ $a_1 \vee (z \leq 3 - \delta) \vee a_2$ $b_1 \vee (z \leq 2 - \delta) \vee b_2$	$\neg a_1 \vee (0 \leq -\delta) \vee \neg a_6$
S_{-z}	$\neg a_6 \vee (-z \leq -5 - \delta)$ $\neg b_6 \vee (-z \leq -4 - \delta)$ $\neg a_1 \vee (-z \leq -3)$ $\neg b_1 \vee (-z \leq -2)$	

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DPFM - Example (on var z)

	OLD	NEW
S_z	$\neg a_1 \vee (z \leq 5)$	$\neg a_1 \vee (0 \leq -\delta) \vee \neg a_6$
	$\neg b_1 \vee (z \leq 4)$	$\neg a_1 \vee (0 \leq 1 - \delta) \vee \neg b_6$
	$a_1 \vee (z \leq 3 - \delta) \vee a_2$	$\neg a_1 \vee (0 \leq 2)$
	$b_1 \vee (z \leq 2 - \delta) \vee b_2$	$\neg a_1 \vee (0 \leq 3) \vee \neg b_1$
S_{-z}	$\neg a_6 \vee (-z \leq -5 - \delta)$	$\neg b_1 \vee (0 \leq -1 - \delta) \vee \neg a_6$
	$\neg b_6 \vee (-z \leq -4 - \delta)$	$\neg b_1 \vee (0 \leq -\delta) \vee \neg b_6$
	$\neg a_1 \vee (-z \leq -3)$	$\neg b_1 \vee (0 \leq 1) \vee \neg a_1$
	$\neg b_1 \vee (-z \leq -2)$	$\neg b_1 \vee (0 \leq 2)$
		$a_1 \vee (0 \leq -2 - \delta) \vee a_2 \vee \neg a_6$
		$a_1 \vee (0 \leq -1 - \delta) \vee a_2 \vee \neg b_6$
		$a_1 \vee (0 \leq -\delta) \vee \neg a_1 \vee a_2$
		$a_1 \vee (0 \leq 1 - \delta) \vee \neg b_1 \vee a_2$
		$b_1 \vee (0 \leq -3 - \delta) \vee b_2 \vee \neg a_6$
		$b_1 \vee (0 \leq -2 - \delta) \vee b_2 \vee \neg b_6$
		$b_1 \vee (0 \leq -1 - \delta) \vee \neg a_1 \vee b_2$
		$b_1 \vee (0 \leq -\delta) \vee \neg b_1 \vee b_2$

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DPFM - Example (on var z)

OLD	NEW
$\neg a_1 \vee (z \leq 5)$	$\neg a_1 \vee \neg a_6$
$\neg b_1 \vee (z \leq 4)$	
$a_1 \vee (z \leq 3 - \delta) \vee a_2$	
$b_1 \vee (z \leq 2 - \delta) \vee b_2$	
$\neg a_6 \vee (-z \leq -5 - \delta)$	$\neg b_1 \vee \neg a_6$
$\neg b_6 \vee (-z \leq -4 - \delta)$	$\neg b_1 \vee \neg b_6$
$\neg a_1 \vee (-z \leq -3)$	
$\neg b_1 \vee (-z \leq -2)$	
	$a_1 \vee a_2 \vee \neg a_6$
	$a_1 \vee a_2 \vee \neg b_6$
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	$b_1 \vee \neg a_1 \vee b_2$

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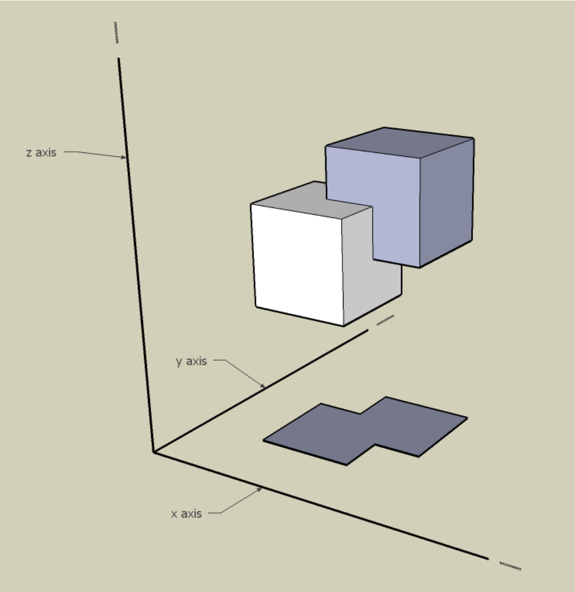

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Variable Elimination - Complexity

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- After elim. a_2 we have $\frac{m^2}{8} \times \frac{m^2}{8} = \frac{m^4}{4^3}$ clauses
- After elim. a_n we have $\frac{m^{2^{n-1}}}{4^{(2^{n-1}-\frac{1}{2})}} \times \frac{m^{2^{n-1}}}{4^{(2^{n-1}-\frac{1}{2})}} = \frac{m^{2^n}}{4^{2^n-1}}$ clauses

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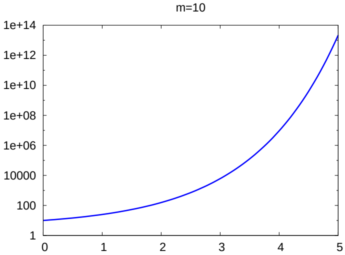
- After elim. a_n we have $\frac{m^{2^{n-1}}}{4^{(2^{n-1}-\frac{1}{2})}} \times \frac{m^{2^{n-1}}}{4^{(2^{n-1}-\frac{1}{2})}} = \frac{m^{2^n}}{4^{2^n-1}}$ clauses



Formula Simplification and Preprocessing

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- **Centrality (for x)**: number of distinct variables that appear in some constraint with x
- **Trade-off (for x)**: amount of new clauses that we want to "trade" for eliminating x

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Formula Simplification - (Centrality 2, Trade-off 128)

OpenSMT on QF_IDL/qlock Benchmarks - Structural Data								
	P.Time (s)		Clauses		TAtoms		TVars	
Bench	WO	W	WO	W	WO	W	WO	W
Ind 37	1.08	6.57	41137	35299	6129	5285	829	185
Ind 38	1.16	6.62	42265	36244	6299	5423	851	188
Ind 39	1.19	7.02	43381	37150	6467	5562	873	189
Ind 40	1.17	7.05	44457	38114	6619	5702	895	203
Base 18	0.80	1.87	18630	16314	2867	2559	375	137
Base 19	0.82	2.31	19780	17269	3045	2702	397	150
Base 20	0.95	2.47	20914	18246	3215	2851	419	151
Base 21	0.94	2.54	22052	19193	3389	2995	441	155

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OpenSMT on QF_IDL/qlock Benchmarks - Solving Time					
Bench	Time WO (s)	Time W (s)	Bench	Time WO (s)	Time W (s)
Base 18	61.3	**59.0**	Ind 37	90.5	**18.0**
Base 19	146.1	**138.4**	Ind 38	105.7	**54.6**
Base 20	> 1800	**940.1**	Ind 39	64.4	**46.7**
Base 21	1367.9	**765.0**	Ind 40	98.3	**37.3**

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Mixed Boolean-Theory Static Learning

OpenSMT on QF_IDL/job_shop/jobshop12-2-6-6-2-4-9.smt							
Centr.	Trade-Off	VE	P.Time	Clauses	TAtoms	BAtoms	T.Time (s)
-	-	0	0.05	216	612	0	> 1800
12	64	0	0.05	216	612	0	> 1800
12	256	2	0.06	458	832	22	180.0
12	1024	4	0.04	1094	968	42	91.4
12	4096	6	0.09	3076	1032	60	67.2
12	16384	6	0.10	3076	1032	60	67.1
18	64	0	0.02	216	612	0	> 1800
18	256	4	0.02	714	1054	56	192.3
18	1024	8	0.07	2005	1566	109	105.6
18	4096	12	0.15	5702	2254	156	125.6
18	16384	12	0.16	5702	2254	156	125.9
24	64	0	0.02	216	612	0	> 1800
24	256	4	0.03	781	1108	66	193.2
24	1024	8	0.07	1978	1638	117	157.1
24	4096	11	0.19	5005	2198	153	89.4
24	16384	12	0.32	5519	2294	163	92.2

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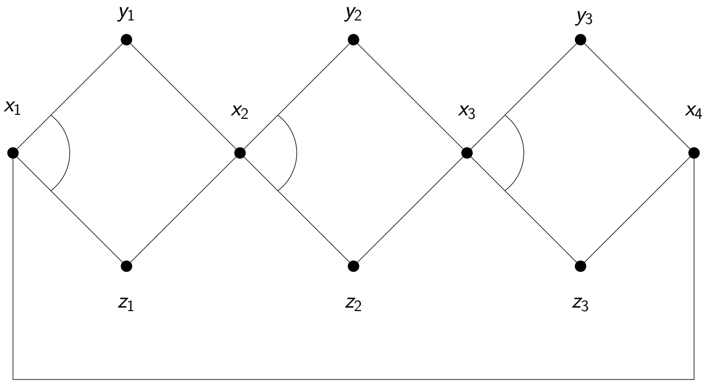
A crazy benchmark suite

Fractal Diamonds

Our preprocessor is effective for those formulæ that are difficult to solve with the initial fixed set of theory atoms



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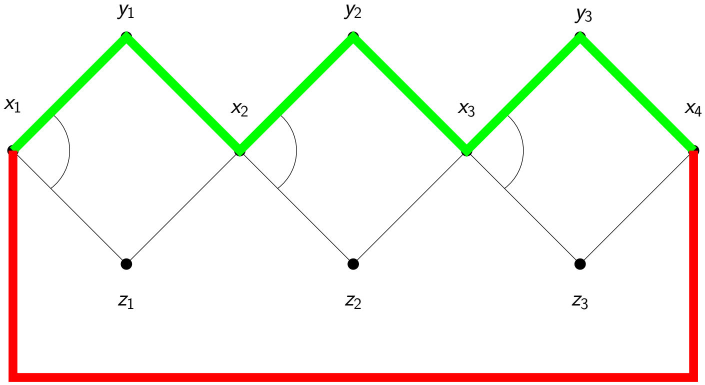

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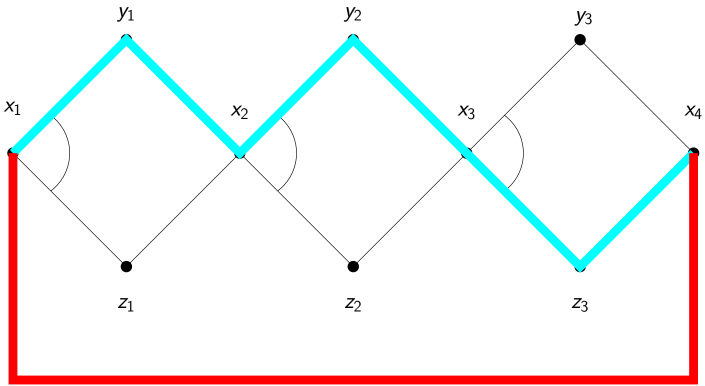

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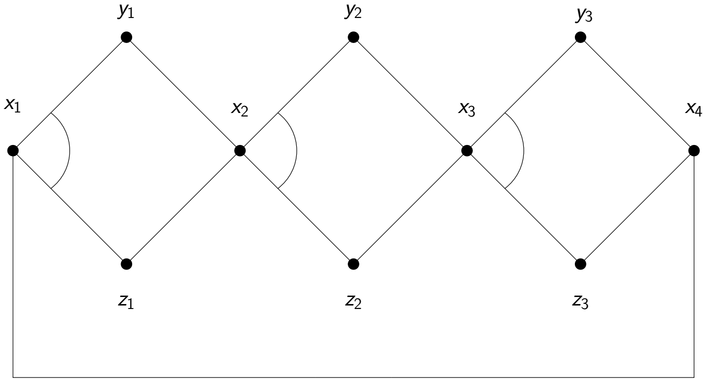

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A crazy benchmark suite
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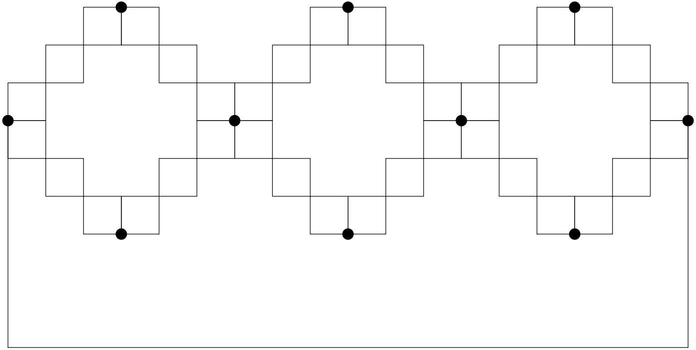
Our preprocessor is effective for those formulæ that are difficult to solve with the initial fixed set of theory atoms



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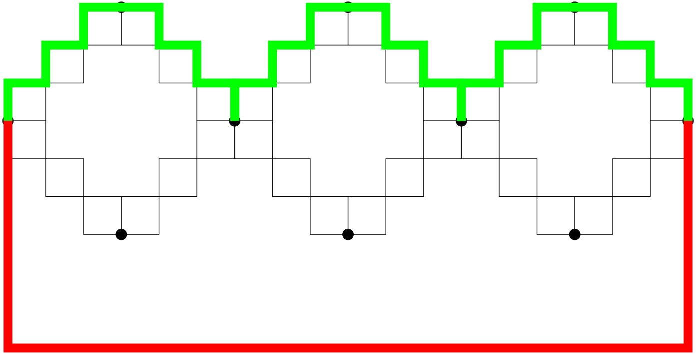

A crazy benchmark suite

Fractal Diamonds (Centrality 18, Trade-off 8192)

B = BarceLogic (SMTCOMP'08 1st place for IDL)
Z = Z3 (SMTCOMP'08 2nd place for IDL)
O = OpenSMT (with DPFM based preprocessor)

Fractal Diamonds - Solving time (s) - TO = 1200 s															
	1			2			3			4			5		
Or.	B	Z	O	B	Z	O	B	Z	O	B	Z	O	B	Z	O

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2	0	0	0	0	0	0	118	13	**1**	T	T	**3**	T	T	**7**

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3	0	0	0	**0**	T	2	T	T	**153**	M	T	T	T	T	T

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Related Work

- O. Strichman: "Deciding Disjunctive Linear Arithmetic with SAT" [Str04]
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- K. McMillan et al.: “Generalizing DPLL to Richer Logics” [MKS09]
  - “Shadow Rule” similar to our notion of SMT($\mathcal{LRA}$) resolution: one application of the shadow rule is equiv. to many applications of SMT($\mathcal{LRA}$) resolution

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Outline

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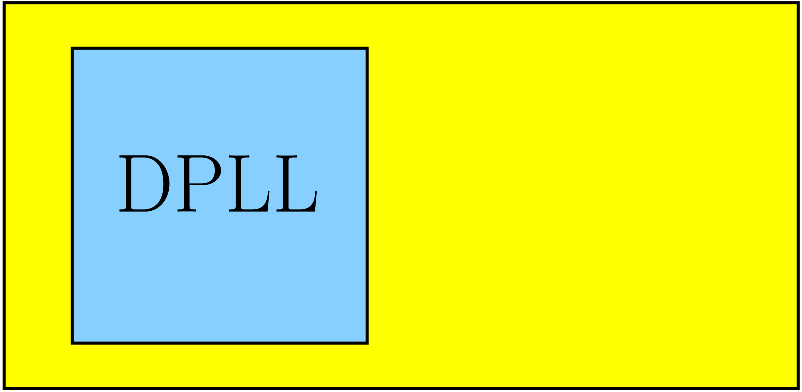
Extending OpenSMT

$$e(\mathrm{DPLL}(T)) \approx \boxed{e(T)}$$



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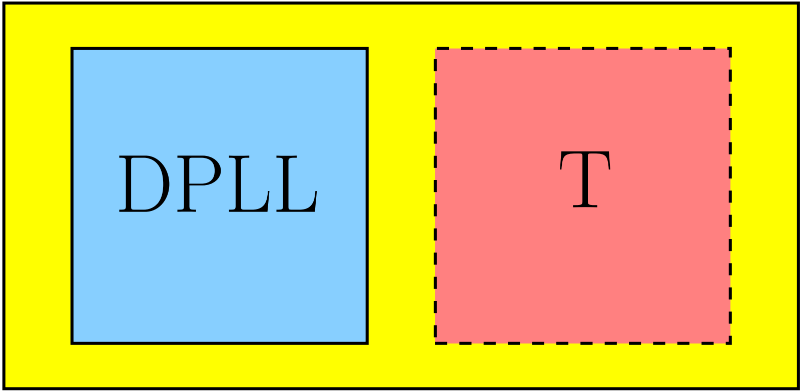

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  - Adds a new logic
  - Integrates the new solver with the core
  - Basically, it creates an incomplete solver

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Extending OpenSMT

```
class TSolver
{
  void   inform         ( Enode * );
  bool   assertLit      ( Enode * );
  bool   check          ( bool );
  void   pushBktPoint   ( );
  void   popBktPoint    ( );
  bool   belongsToT     ( Enode * );
  void   computeModel   ( );

  vector< Enode * > & explanation;
  vector< Enode * > & deductions;
  vector< Enode * > & suggestions;
}
```

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- Basically I is an "overapproximation" of A, that is still unsatisfiable with B
- It has applications in Model Checking [McM04]
- OpenSMT can compute interpolants for propositional formulæ and some arithmetic fragments

Conclusion

- OpenSMT website
  http://www.verify.inf.unisi.ch/opensmt
- Source repository
  http://code.google.com/p/opensmt
- Discussion group
  http://groups.google.com/group/opensmt

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