Synchronizing Finite Automata

IX. Open Problems

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1. Recap

Deterministic finite automata: $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$.

- Q the state set
- Σ the input alphabet
- $\delta : Q \times \Sigma \to Q$ the transition function

$\mathscr{A}$ is called synchronizing if there exists a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is, leaves the automaton in one particular state no matter which state in Q it started at: $\delta(q, w) = \delta(q', w)$ for all $q, q' \in Q$.

$|Q \,.\, w| = 1$. Here $Q \,.\, v = \{\delta(q, v) \mid q \in Q\}$.

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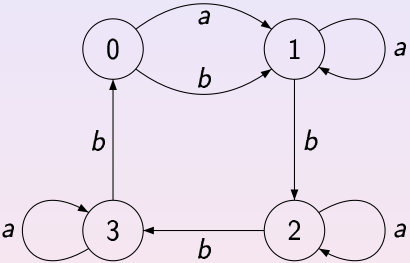

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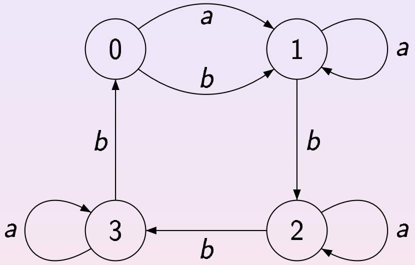

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3. Classical Problems

- The Černý Conjecture: is it true that, for any synchronizing automaton with n states, there exists a reset word of length $(n-1)^2$?
- The Rank Conjecture: is it true that, for any DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ with n states and rank k, there is a $w \in \Sigma^*$ of length $(n-k)^2$ such that $|\delta(Q, w)| = k$?

The rank of a DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is the minimum cardinality of the sets $Q\,.\,w$ where w runs over Σ^*. This is the minimum score that can be achieved in the solitaire game on the automaton $\mathscr{A}$. Synchronizing automata are precisely those of rank 1.

- The Černý function's behaviour for various restricted classes of synchronizing automata.

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Example: Recently Trahtman has proved that every synchronizing aperiodic automaton with n states admits a reset word of length at most $\frac{n(n-1)}{2}$.

However no precise bound for $C_A(n)$, the minimum length of reset words for synchronizing aperiodic automata with n states, has been found so far.

(Trahtman) $\frac{n(n-1)}{2} \geq C_A(n) \geq n + \lfloor \frac{n}{2} \rfloor - 2$ (Ananichev)

The gap between the upper and the lower bounds is rather drastic.

Similar things happen for several other classes of synchronizing automata for whose Černý functions we know an upper bound (Eulerian, weakly monotonic).

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5. Letters of Deficiency 2

Let $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ be a DFA with $|Q| > 3$. The deficiency of a word $v \in \Sigma^*$ w.r.t. $\mathscr{A}$ is the difference $\mathrm{df}_{\mathscr{A}}(v) = |Q| - |\delta(Q, v)|$. If a letter $a \in \Sigma$ has deficiency 2, then exactly one of the two following situations happens:

1. There are three different states $q_1, q_2, q_3 \in Q$ such that

$$\delta(q_1, a) = \delta(q_2, a) = \delta(q_3, a).$$

2. There are four different states $q_1, q_2, q_3, q_4 \in Q$ such that

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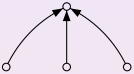
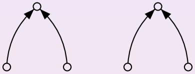
In the first situation we say that a is a dromedary letter, and in the second case we say that a is a bactrian letter.



The action of a dromedary letter



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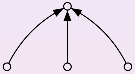

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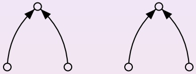

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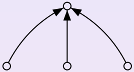

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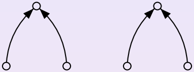

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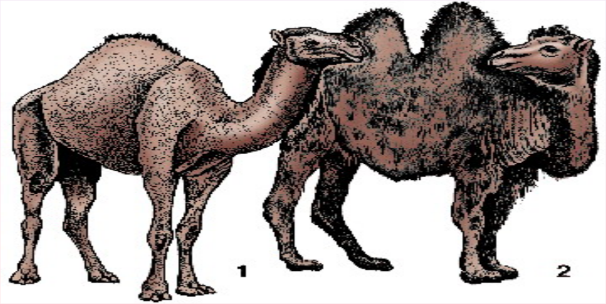

7. Letters of Deficiency 2

For each $n > 4$, there is a synchronizing automaton with n states and 3 input letters one of which is dromedary whose shortest reset word is of length $(n-2)^2 + 1$.

For each odd $n > 3$, there is a synchronizing automaton with n states and 2 input letters one of which is bactrian whose shortest reset word is of length $(n-1)(n-2)$.

Do these lower bounds represent the worst possible case? In other words, is the Černý function for the class of synchronizing automata with a dromedary/bactrian letter equal to $(n-2)^2 + 1$/respectively $(n-1)(n-2)$?

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For each $n > k + 1$ not divisible by k, there is a synchronizing automaton with n states and 2 input letters one of which has deficiency k whose shortest reset word is of length $(n-1)(n-k)$.

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Observe that again the original Černý conjecture correspond to the case $k = 1$.

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The hybrid Černý/Road Coloring problem: if a strongly connected primitive digraph with constant out-degree has n vertices, what is the minimum length of reset words for its synchronizing colorings?

For instance, the Černý automata admit synchronizing recolorings with pretty short reset words.

The recolored automaton is reset by the word b^{n-1}.

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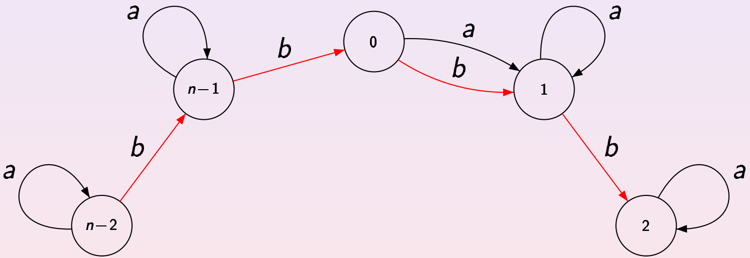

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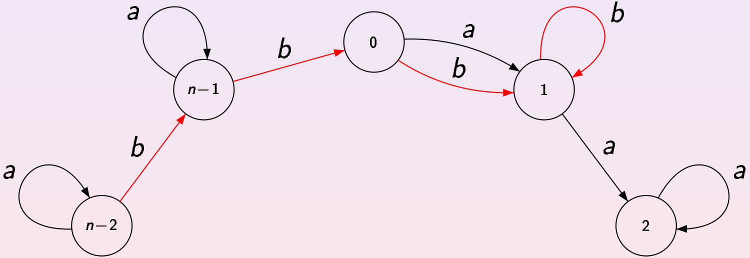

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Problems on Synchronizing Digraphs

We have the lower bound $n^2 - 3n + 3$ (Wielandt automata) and conjecture that it is tight.

Characterize totally synchronizing digraphs Γ, that is, such that each coloring of Γ is synchronizing. For instance, underlying digraphs of the Černý automata are totally synchronizing, and so are Wielandt's digraphs yielding the lower bound in the previous problem.

Universal reset words for a given digraph: we say that a word w is a universal reset word for Γ if it is a reset word for every synchronizing coloring of Γ. If Γ has n vertices, what is the minimum length of universal reset words for Γ?

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- From the viewpoint of transportation network the constant out-degree condition does not seem to be natural. We rather want to find a synchronizing coloring for an arbitrary strongly connected primitive digraph Γ, the number of colors being the maximal out-degree of Γ.

But in the absence of the constant out-degree condition, the resulting automaton $\mathscr{A} = \langle \mathbf{Q}, \mathbf{\Sigma}, \delta \rangle$ is incomplete. We need a suitable modification of the notion of a synchronizing automaton for this case.

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But in the absence of the constant out-degree condition, the resulting automaton $\mathscr{A} = \langle \mathbf{Q}, \boldsymbol{\Sigma}, \delta \rangle$ is incomplete. We need a suitable modification of the notion of a synchronizing automaton for this case.

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10. Careful Synchronization

We say that $w = a_1 \cdots a_\ell$ with $a_1, \dots, a_\ell \in \Sigma$ is a *careful reset word* for $\mathscr{A}$ if

- $\delta(q, a_1)$ is defined for all $q \in Q$,
- $\delta(q, a_i)$ with $1 < i \leq \ell$ is defined for all
$$q \in Q \,.\, a_1 \cdots a_{i-1},$$
- $|Q \,.\, w| = 1$.

Careful synchronization has been much studied recently and has proved to be more complicated than the usual synchronization. The minimum length of careful reset words may be exponential of the number of states, checking that an automaton is carefully synchronizing is PSPACE-complete.

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In the transport network terms careful synchronization means that there is an instruction which is always possible to follow and which brings one to the node which is independent of the initial node.

Careful Road Coloring Problem: under which conditions does a strongly connected digraph Γ admit a carefully synchronizing coloring?

Primitivity is still necessary but it is not sufficient anymore. The next slide presents an example of a strongly connected primitive digraph which admits no carefully synchronizing coloring.

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